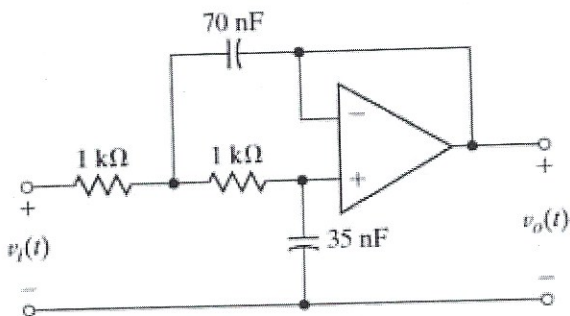


Solution

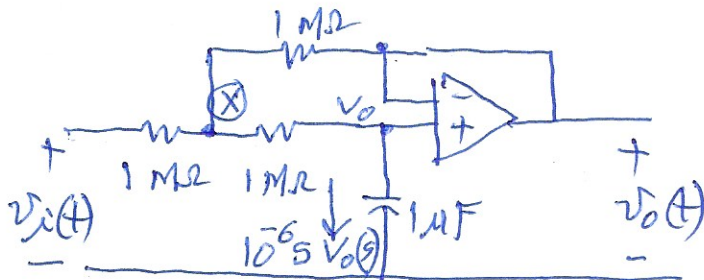
Name _____ ID _____

This problem is to test your ability for finding $H(s)$ and output voltage for a given input voltage by using the Laplace Transform method.

Let us modify the same circuit you have studied below:



(a). (2 points) Redraw the circuit by changing $1\text{ k}\Omega$ resistors to $1\text{ M}\Omega$ resistors and 35 nF capacitor to $1\text{ }\mu\text{F}$ and 70 nF capacitor to $1\text{ M}\Omega$. (The circuit becomes a first order filter with only one capacitor.)



(b). (4 points) Find $H(s) = V_o(s)/V_i(s)$

$$V_i(s) = 10^6 [10^{-6} s V_o + 10^6 (V_{\text{X}} - V_o)] + V_{\text{X}} \quad (1)$$

$$V_{\text{X}} = 10^6 \times 10^{-6} s V_o(s) + V_o \quad (2)$$

From (1) & (2)

$$\begin{aligned} V_i(s) &= s V_o + s V_o(s) + s V_o(s) + V_o(s) \\ &= (3s + 1) V_o(s) \end{aligned}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{3s + 1} = \frac{\frac{1}{3}}{s + \frac{1}{3}} = H(s)$$

(ms)

(c). (4 points) Find $v_o(t)$ for a step input $u(t)$. Assume zero initial condition.

$$V_o(s) = H(s) \cdot U(s) = \frac{\frac{1}{3}}{s + \frac{1}{3}} \cdot \frac{1}{s}$$
$$\mathcal{L}u(t) = \frac{1}{s} = \frac{-1}{s + \frac{1}{3}} + \frac{1}{s}$$

$$v_o(t) = (1 - e^{-\frac{1}{3}t}) u(t) \quad \text{ans}$$