(a) ①

(1) The I/P is a square wave.

(2) From 4.3, it is given for even K F.S coefficients = 0

for odd K $C_K = \frac{-j2X_o}{\pi K}$, where X_o = 10.

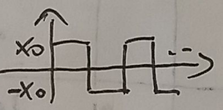
(3) Hence the first three nonzero I/P harmonics are as follows

$C_1 = \frac{-20j}{\pi}$, $C_3 = \frac{-20}{3\pi}j$, $C_5 = \frac{-20j}{5\pi}$, $C_{-1} = \frac{20j}{\pi}$, $C_{-3} = \frac{20j}{3\pi}$, $C_{-5} = \frac{20j}{5\pi}$

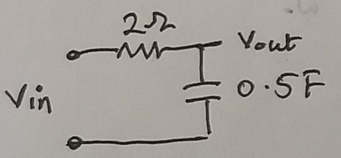
(4) Calculate system Transfer function. This will help us evaluate corresponding system coeffients.

(5) Multiply I/P & system coefficients to get O/P coefficients

input = square wave.



where $X_o = 10V$



$V_{out} = \frac{Z_c}{Z_c + R} \times V_{in}$.

$$\Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{Z_c}{Z_c + R}}$$

$Z_c = \frac{1}{j\omega C}$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{1/j\omega C}{1/j\omega C + R}.$$

$$= \frac{1/j\omega C}{\frac{1 + j\omega RC}{j\omega C}}$$

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$R = 2\Omega$, $C = \frac{1}{2}F$

$$\Rightarrow \boxed{H(j\omega) = \frac{1}{1 + j\omega}}$$

∴ The system coefficients are:

$$H(j1) = \frac{1}{1+j}, \quad H(j3) = \frac{1}{1+3j}, \quad H(j5) = \frac{1}{1+5j}$$

$$= \frac{1-j}{2} \qquad = \frac{1-3j}{10} \qquad = \frac{1-5j}{26}$$

$$H(-j) = \frac{1+j}{2} \qquad H(-j3) = \frac{1+3j}{10} \qquad H(-j5) = \frac{1+5j}{26}.$$

For o/p coefficient

$$\omega_0 = \frac{2\pi}{T_0} \qquad T_0 = 2\pi$$

$$= \frac{2\pi}{2\pi} = 1 \text{ rad/s.}$$

$$C_1 = C_{in\,1} \times H(jk\omega_0)$$

$$= \frac{-20j}{\pi} \cdot \frac{1-j}{2}$$

$$\boxed{= \frac{-10}{\pi}(1+j)} \Rightarrow 4.50 \angle -135^\circ$$

$$C_3 = \frac{-20j}{3\pi} \frac{(1-3j)}{10}$$

$$\boxed{= \frac{-2}{3\pi}(1+3j)} \Rightarrow 0.67 \angle -108^\circ$$

$$C_5 = \frac{-20j}{5\pi} \frac{(1-5j)}{26}$$

$$\boxed{= \frac{-2}{13\pi}(1+5j)} \Rightarrow 0.25 \angle -101^\circ$$

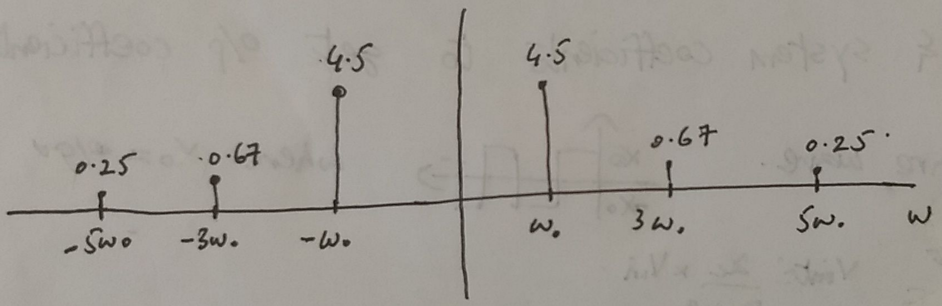
$$C_1 = \frac{20j}{\pi} \times \frac{(1+j)}{2}$$

$$= \frac{10}{\pi}(-1+j) = \frac{-10}{\pi}(1-j) = 4.5\angle 135^\circ$$

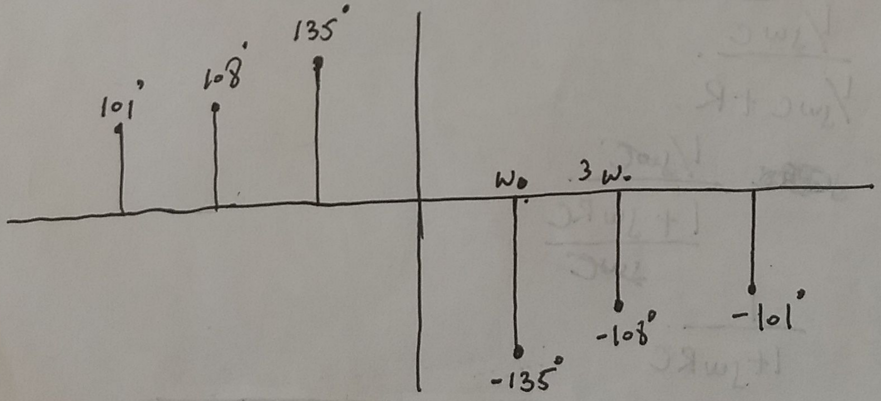
$$C_{-3} = \frac{-2}{3\pi}(1-3j) = 0.67\angle 108^\circ$$

$$C_{-5} = \frac{-2}{13\pi}(1-5j) = 0.25\angle 101^\circ$$

Amp Response.



Phase Response.



(c) periodic function $x(t)$ is repersented as.

$$x(t) = A_0 + \sum_{k=1}^{\infty} C_k e^{j\omega_0 k t}$$

The DC value will have no effect on the first 3 harmonics. They remain the same as in part 1.

As for the DC component

$C_0 = H(0)\, C_{in\,0}$

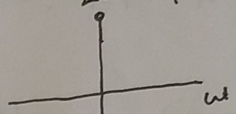
$\boxed{= 20V.}$

$C_{in\,0} = 20V.$

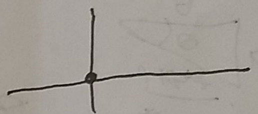
$H(j\omega) = \frac{1}{1+j\omega}$

$H(0) = 1$

20 Amp Response



Phase Response



(d) There are multiple ways to ans this.

$H(j\omega) = \frac{1}{1+j\omega}$

For $\omega >> 1$.

$1+j\omega \approx j\omega$.

$\therefore\ H(j\omega) \approx \frac{1}{j\omega}$

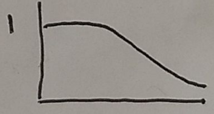
For higher values of 'w' $H(j\omega) \Rightarrow 0$

For $\omega << 1$.

$1+j\omega \approx 1$.

$H(j\omega) = 1$

$\longrightarrow$.



Hence a: Low pass filter.

e) The fundamentle frequency has changed from 1rad/s → 2rad
this will change the distance between the harmonics

(2) I/p signal

$$x(t) = \sum_{K=1}^{\infty} \cos(Kt)$$

Impulse Response of system.

$$h(t) = e^{-at} u(t)$$

From Table 4.5, steady state sinusoidal response for LTI system with I/p $x(t) = X\cos(\omega_1 t + \phi)$

$$|X|\,|H(j\omega_1)|\cos(\omega_1 t + \phi + \angle H(j\omega_1)) \quad \text{———} \quad 4.41$$

$$\boxed{\text{Here} \quad X = 1 \quad \omega = K \quad \phi = 0.}$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}\,dt.$$

$$= \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-j\omega t}\,dt.$$

$$= \cancel{\int_{-\infty}^{0} e^{-t(a+j\omega)} u(t)}^{\to 0} + \int_{0}^{\infty} e^{-t(a+j\omega)} u(t)\,dt.$$

$$= \int_{0}^{\infty} e^{-t(a+j\omega)}\,dt.$$

$$\boxed{H(j\omega) = \frac{1}{j\omega + a}.}$$

According to 4.41. And $\boxed{X=1,\ \omega=K,\ t=0}$

$$Y(t) = \sum_{K=1}^{\infty} |H(jK)| \cos(Kt + \angle H(jK)),$$

For $H(jK) = \frac{1}{jK+a}$

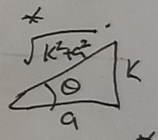
$$|H(jK)| = \frac{1}{\sqrt{K^2+a^2}}$$

$$\angle H(jK) = \tan^{-1}\left(-\frac{K}{a}\right)$$

$$Y(t) = \sum_{K=1}^{\infty} \frac{1}{\sqrt{K^2+a^2}} \cdot \cos\left(Kt + \tan^{-1}\left(-\frac{K}{a}\right)\right)$$

$$= \sum_{K=1}^{\infty} \frac{1}{\sqrt{K^2+a^2}} \cos\left(Kt - \tan^{-1}(K/a)\right)$$

$$\Rightarrow \cos(A \mp B) = \cos A \cos B \pm \sin A \sin B.$$



$$= \sum_{K=1}^{\infty} \frac{1}{\sqrt{K^2+a^2}} \cos Kt \cos\left(\tan^{-1}\left(\frac{K}{a}\right)\right)^{*} + \sin Kt\left(\sin\left(\tan^{-1}\frac{K}{a}\right)\right)^{*}$$

$$= \sum_{K=1}^{\infty} \frac{1}{\sqrt{K^2+a^2}} \cos Kt \frac{a}{\sqrt{a^2+K^2}} + \sin Kt \frac{K}{\sqrt{a^2+K^2}}$$

$$\boxed{Y(t) = \sum_{K=1}^{\infty} \frac{a\cos(Kt) + K\sin(Kt)}{a^2+K^2}}$$