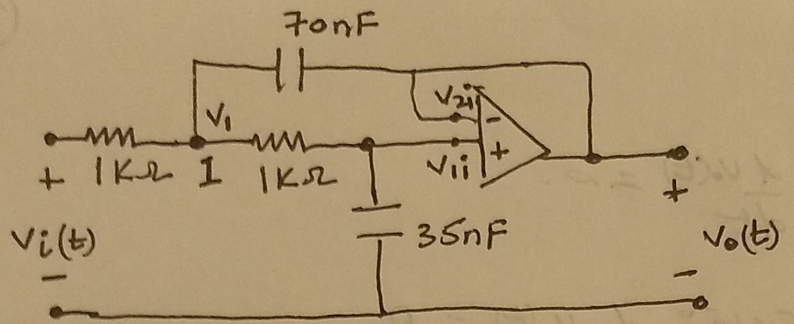
(1) 

①

The above op-amp is ideal

$\therefore \quad I_i = 0$ $\qquad$ I_i - input current to Op-Amp

$V_{1i} = V_{2i}$ $\qquad$ V_1, V_2 - The two i/p voltage to Op-Amp

V_1 - Voltage at Node 1

(a) Find $\dfrac{V_o(\omega)}{V_i(\omega)} = H(\omega)$

To Find relationship b/w $V_i(t)$ & $V_o(t)$ apply KCL at node 1

$$V_{2i}(t) = V_{1i}(t) = V_o(t).$$

$$\frac{V_1(t) - V_i(t)}{1K} + \frac{V_1(t) - V_o(t)}{1K} + 70n\,\frac{d(V_1(t) - V_o(t))}{dt} = 0$$

$$\frac{2V_1(t) - V_i(t) - V_o(t)}{1K} + \frac{70}{10^9}\frac{d}{dt}(V_1(t) - V_o(t)) = 0.$$

$$V_1(t)\left(2 + 70\times10^{-6}\frac{d}{dt}\right) - V_o(t)\left(1 + 70\times10^{-6}\frac{d}{dt}\right) - V_i(t) = 0$$

Take FT

$$V_1(\omega)(2 + j\omega\,70\times10^{-6}) - V_o(\omega)(1 + j\omega\,70\times10^{-6}) - V_i(\omega) = 0 \quad \text{—①}$$

Take KCL at V_i ②

$$\frac{V_o(t) - V_1(t)}{1K} + 35n \frac{d\,V_o(t)}{dt} = 0.$$

$$V_o(t) - V_1(t) + 35\times10^{-6} \frac{d}{dt} V_o(t) = 0$$

Tak FT.

$$V_o(\omega) - V_1(\omega) + V_o(\omega) j\omega(35\times10^{-6}) = 0$$

$$V_1 = V_o(1 + j\omega(35\times10^{-6})) \quad - ②$$

Plug ② in ①.

$$V_o(1+j\omega(35\times10^{-6}))(2+j\omega\,70\times10^{-6}) - (1+j\omega\,70\times10^{-6})V_o = V_{i}(t)$$

$$\frac{V_o}{V_i} = H(\omega) = \frac{1}{(2 + j\omega\,\underline{70\times10^{-6}} + \cancel{j\omega\,70\times10^{-6}} - 2\omega^2(35\times10^{-6})^2 - 1 - \cancel{j\omega\,70\times10^{-6}}}$$

$$= \frac{1}{1 + j\omega\,70\times10^{-6} - 2(\omega\,35\times10^{-6})^2}$$

it would have been more clear if 1K = R. 75nF = C_1 35nF = C_2

The underline 70×10^{-6} is infact $2C_2 = 70\times10^{-6}$ & if we had not cancelled 1K. The resulting expression would have been in terms of RC. which would help in part (b).

(b) As given in the book.

The -3dB bandwidth/freq is when

$$\frac{P_o}{P_1} = \frac{1}{2}$$

Take that in dB scale.

$$10 \lg\left[\frac{P_o}{P_1}\right] = 10 \lg\left(\frac{1}{2}\right) = -3\,dB.$$

The freq at this point is called

Corner freq 'ω_c'. For this 2nd order Butterworth

$$\omega_c = \frac{1}{\sqrt{2} R C_2} \simeq 20\,200 \text{ rad/sec.}$$

(2)
(a) Acc. to Shannon's Sampling theorm to avoid aliasing

$\omega_s \geq 2\omega_M$ where ω_s - Sampling Freq.

ω_M - highest Freq Comp. of signal

As we Multiply $x(t)$ with an impulse train, clearly we are sampling $x(t) \rightarrow x_p(t)$ and then reconstruct it as $y(t)$

$$\omega_s \geq 2\omega_M$$

$$\frac{2\pi}{T} \geq 2\omega_M.$$

$$T \leq \frac{\pi}{\omega_m}$$

(b) For $y(t) = x(t)$ $x_p(t)$ must past through a low pass with filter ~~___~~

$$H(\omega) = T \,\mathrm{rect}(\omega/\omega_s)$$

(5)

(c)

For $T=0.004 sec.$

$\boxed{\begin{array}{l} f < 700Hz \\ \omega < 1400\pi \end{array}}$

$$\omega_s = \frac{2\pi}{T_s} = \frac{2\pi}{0.004} = 500\pi$$

$$x(t) = \cos(200\pi t)$$

$$p(t) = \sum_{K=-\infty}^{\infty} \delta(t-KT)$$

$$x_p(t) = x(t) \times p(t).$$

$$X(\omega) = \pi\left(\delta(\omega-200\pi) + \delta(\omega+200\pi)\right)$$

$$P(\omega) = \omega_s \sum_{K=-\infty} \delta(\omega - 4\omega_s)$$

$$X_p(\omega) = \mathcal{F}\left[x(t)\, p(t)\right]$$

$$= \frac{1}{2\pi} X(\omega) * P(\omega)$$

$$= \frac{\omega_s}{2\pi}\left(\sum_{K=-\infty}^{\infty}\left[\pi\left(\delta(\omega-200\pi)+\delta(\omega+200\pi)\right)\right] * \delta(\omega-K\omega_s)\right)$$

$$= \frac{\omega_s}{2}\sum_{K=-\infty}^{\infty}\left[\delta(\omega-200\pi-\omega_s K) + \delta(\omega+200\pi-\omega_s K)\right.$$

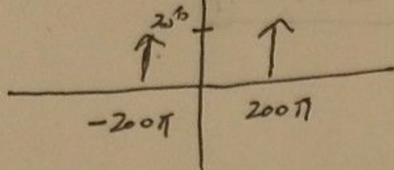
As, $\omega_s = 500\pi$

$$X_p(\omega) = 250\sum_{K=-\infty}^{\infty}\left[\delta(\omega-200\pi-500\pi K) + \delta(\omega+200\pi-500\pi K)\right.$$

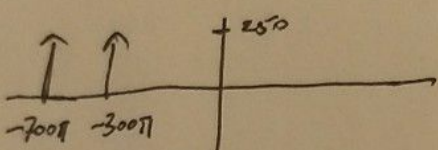
6

Plot for $K = 0, \pm 1, \pm 2 \ldots$ for $\omega < 1400\pi$

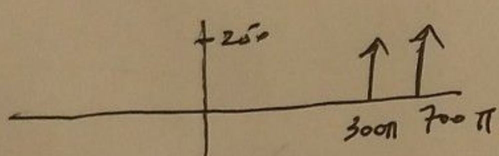
$K=0$



$K=1$



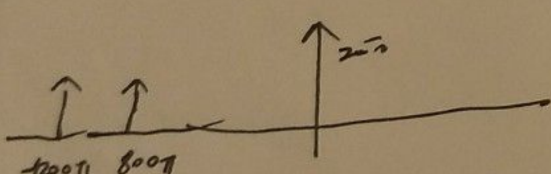
$K=-1$



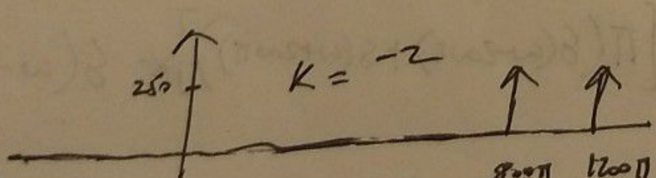
freq components below 1400π are.

$$\boxed{\pm 200\pi, \pm 300\pi, \pm 700\pi, \pm 800\pi, \pm 1200\pi, \pm 1300\pi.}$$

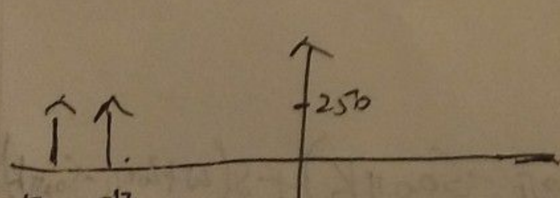
$K=+2$



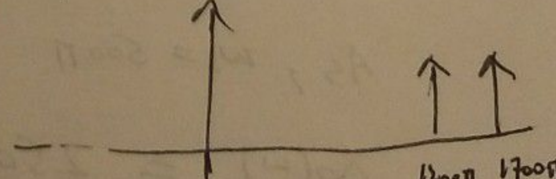
$K=-2$



$K=+3$



$K=-3$



(d) $x(t) = \cos(2\pi f_x t)$

In part c $f_x = 100\,Hz$ $\quad \omega = 200\pi$

$\omega_s = 500\pi$

we need to find ω such that it has the same frequences

① Notice the lowest freq comp $\pm 200\pi \overset{\text{For } K=1}{=} \mp(500\pi(1) - 300\pi)$

② similarly the sec. freq Comp $\pm 300\pi \overset{\text{For } K=0}{=} \mp(500\pi(0) - 300\pi)$

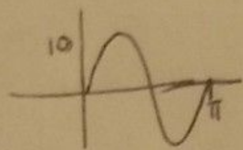
This is only possible for $\omega = 300\pi$

$x(t) = \cos(300\pi t)$

Notice $K \pm 2$ & ± 3 you will ~~get~~ the other components.

(3)

(a)

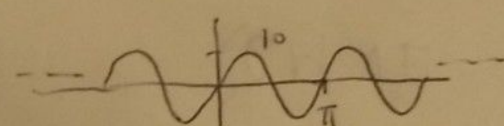


The above wave form is an ideal sine wave that exists for only one cycle. With Amp = 10 & Period = π sec with $\phi = 0$. $\omega = 2\pi f$ $f = \frac{1}{\pi}$

$\boxed{\omega = 2}$

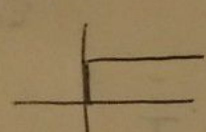
Normal sine wave with above paramiter from $-\infty$ to ∞

$10 \sin(2t)$

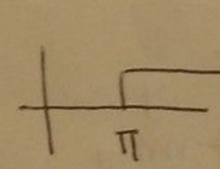


To limit sine wave from 0 — π we use the concept of unit step function.

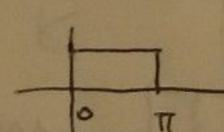
$u(t) \rightarrow$



$u(t-\pi) \rightarrow$



$[u(t) - u(t-\pi)] \longrightarrow$



This is one in our area of interest & 0 everywhere else.

$$\therefore \boxed{f(t) = 10 \sin(2t) [u(t) - u(t-\pi)]}$$

(b) $\mathcal{L}[f(t)] = F(s)$ 9

$$F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} 10 \sin(2t)\, [u(t) - u(t-\pi)]\, e^{-st}\, dt$$

If we change the integral limits from $0 - \pi$ we can express the above expression as:

$$F(s) = \int_{0}^{\pi} 10 \sin(2t)\, e^{-st}\, dt$$

From Integration table.

$$\int e^{xt} \sin(yt)\, dt = \frac{1}{x^2+y^2}\left[e^{xt}\left(x\sin(yt) - y\cos(yt)\right)\right] + C$$

$$\therefore \quad F(s) = \frac{10}{s^2+4}\left[e^{-st}\left(-s\sin(2t) - 2\cos(2t)\right)\right]\Big|_0^{\pi}$$

$$= \frac{10}{s^2+4}\left[\left[e^{-s\pi}(-s(0) - 2(1))\right] - \left[e^{-s(0)}(-s(0) - 2(1))\right]\right]$$

$$= \frac{10}{s^2+4}\left(-2e^{-s\pi} + 2\right)$$

$$= \frac{20}{s^2+4}\left(1 - e^{-s\pi}\right)$$

(c) $f(t) = 10\sin(2t)[u(t) - u(t-\pi)]$ (10)

$$f(t) = 10\sin(2t)(u(t)) - 10\sin 2(t-\pi)\, u(t-\pi)$$

$$\mathcal{L}[f(t)] = F(s)$$

$$F(s) = \mathcal{L}[10\sin(2t)(u(t))] - \mathcal{L}[10\sin 2(t-\pi)\, u(t-\pi)]$$

$$= 10\int_{-\infty}^{\infty} \sin 2t\, u(t)\, dt - 10\int_{-\infty}^{\infty} \sin 2(t-\pi)\, u(t-\pi)\, dt$$

$$= 10\int_{0}^{\infty} \sin 2t\, dt - 10\int_{-\infty}^{\infty} \sin 2(t-\pi)\, u(t-\pi)\, dt$$

From table 7.2 Property 10 : $\boxed{\mathcal{L}[\sin(bt)] = \frac{b}{s^2+b^2}}$ (1)

From Eq 7.21 (Real shifting) to 7.22

$$\mathcal{L}[f(t-t_0)u(t-t_0)] = \int_0^{\infty} f(\tau)\, e^{-s(\tau+t_0)}\, d\tau \quad \text{where } \tau = t - t_0$$

$$= e^{-t_0 s}\int_0^{\infty} f(\tau)\, e^{-s\tau}\, d\tau$$

$$\therefore \boxed{\mathcal{L}[f(t-t_0)\, u(t-t_0)] = e^{-t_0 s} F(s)} \quad (2)$$

using (1) & (2)
$\Rightarrow$

$$F(s) = 10\left[\frac{2}{s^2+4}\right] - 10e^{-\pi s}\left(\frac{2}{s^2+4}\right)$$

$$= \frac{20(1-e^{-\pi s})}{s^2+4}$$