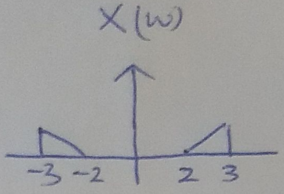
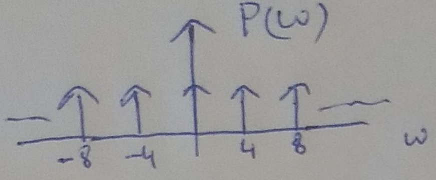
(1) $\hat{x}(t) = x(t)\,p(t)$

$$P(\omega) = 4\sum_{K=-\infty}^{\infty} \delta(\omega - 4K)$$





Find FT of $\hat{x}(t)$

$\hat{x}(t) = x(t)\,p(t)$.

$$\hat{X}(\omega) = \mathcal{F}[x(t)\,p(t)]$$

$$= \frac{1}{2\pi} X(\omega) * P(\omega)$$

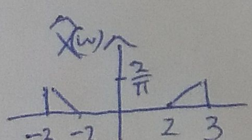
$$= \frac{1}{2\pi} X(\omega) * 4\sum_{K=-\infty}^{\infty} \delta(\omega - 4K)$$

$$= \frac{2}{\pi}\sum_{K=-\infty}^{\infty} X(\omega) * \delta(\omega - 4K)$$

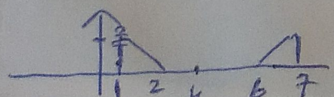
using Property $\delta(\omega - \omega_0) * g(\omega) = g(\omega - \omega_0)$.

$$\hat{X}(\omega) = \frac{2}{\pi}\sum_{K=-\infty}^{\infty} X(\omega - 4K)$$

For $K = 0$, $\hat{X}(\omega) = \frac{2}{\pi} X(\omega)$.

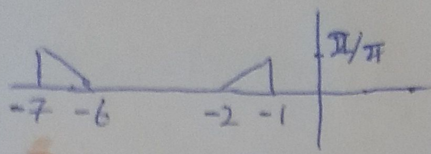


For $K = 1$, $\hat{X}(\omega) = \frac{2}{\pi}(X(\omega - 4))$

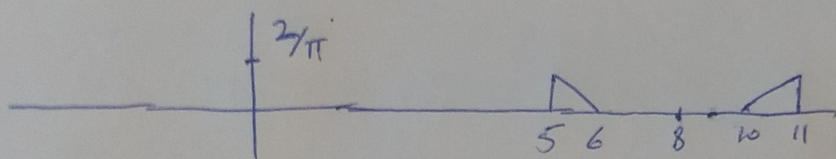


For $K = -1$

$$\hat{X}(\omega) = \frac{2}{\pi}\left(X(\omega+4)\right)$$

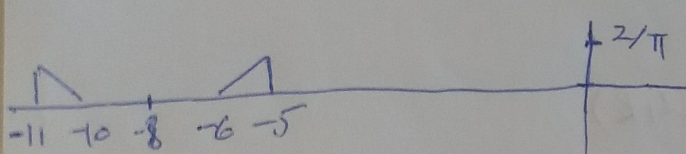


For $K = 2$, $\hat{X} = \frac{2}{\pi}\left(X(\omega-8)\right)$

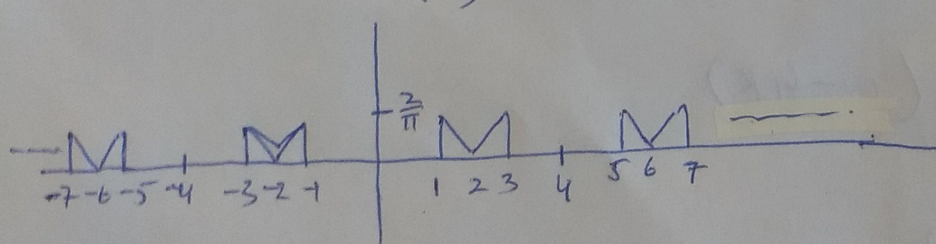


For $K = -2$.

$$\hat{X} = \frac{2}{\pi}\left(X(\omega+8)\right)$$

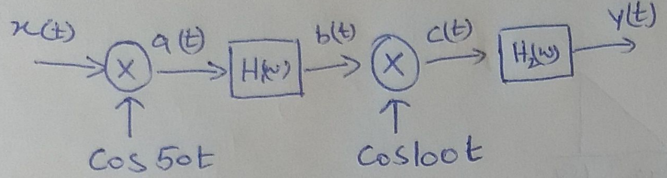


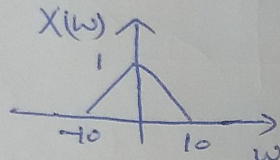
$\hat{X}(\omega)$



(2)

6.27

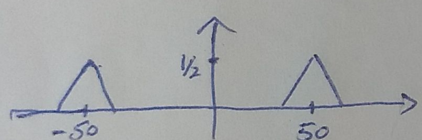




$a(t) = x(t) \times \cos 50t$

$= x(t)\left[\frac{e^{j50t} + e^{-j50t}}{2}\right]$

$A(w) = \frac{1}{2}\left[X(w-50) + X(w+50)\right]$



OR

$f(x(t)) \rightarrow X(w)$.

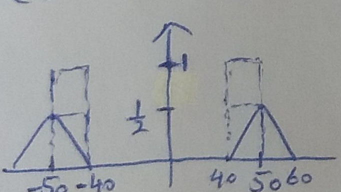
$f[\cos 50t] \rightarrow \pi[\delta(w-50) + \delta(w+50)]$

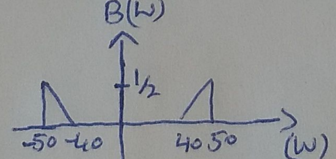
$f[x(t)\cos 50t] = \frac{1}{2\pi}(X(w) * \pi[\delta(w-50) + \delta(w-50)])$

$= \frac{1}{2}[(X(w) * \delta(w-50)) + (X(w) * \delta(w+50))]$

$= \frac{1}{2}\left[X(w-50) + X(w+50)\right]$

$B(w) = A(w)\ H_1(w)$

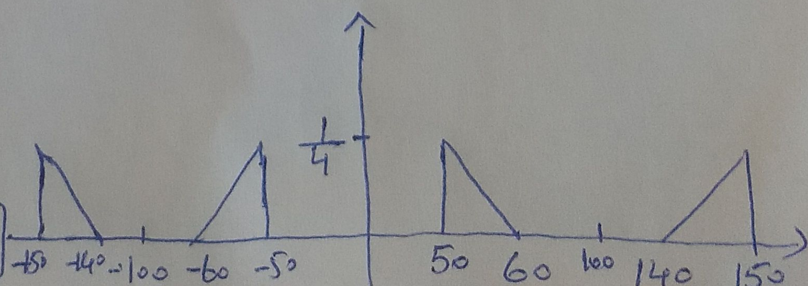




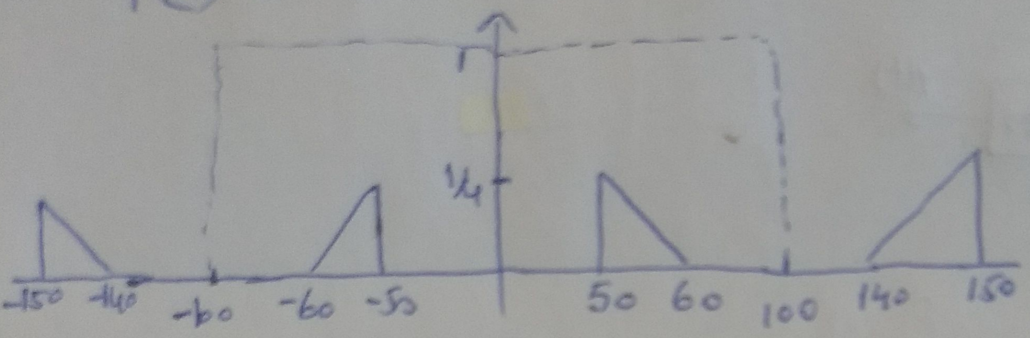
$c(t) = b(t) \cos 100t$

$= b(t)\left[\frac{e^{j100t} + e^{-j100t}}{2}\right]$

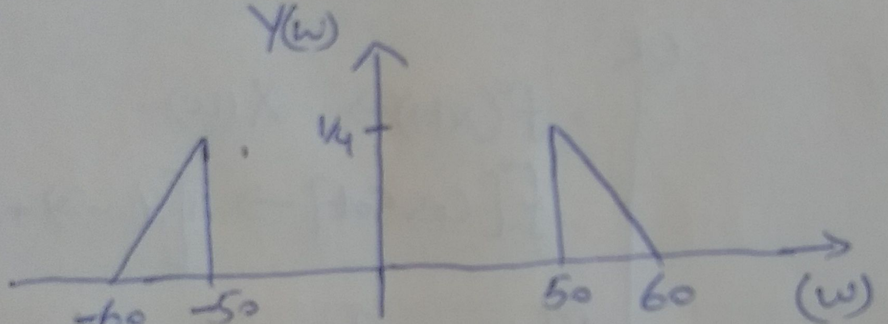
$C(w) = \frac{1}{2}\left[B(w-50) + B(w+50)\right]$



$$Y(\omega) = C(\omega)\, H_2(\omega)$$



$$\Downarrow$$



(3)

(a) $g_1(t) = \phi(t) \cos w_c t$

$\phi(t) = f_1 \cos w_c t + f_2 \sin w_c t$.

$g_1(t) = (f_1 \cos w_c t + f_2 \sin w_c t) \cos w_c t$.

$= f_1 \cos^2 w_c t + f_2 \sin w_c t \cos w_c t$.

we know.

$2 \sin \alpha \cos \alpha = \sin 2\alpha$.

$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$.

$\Rightarrow g_1(t) = f_1 \left(\frac{1 + \cos 2w_c t}{2}\right) + \frac{f_2}{2} \sin 2 w_c t$.

$= \frac{f_1}{2} + \frac{f_1 \cos 2 w_c t}{2} + \frac{f_2 \sin 2 w_c t}{2}$

(b) $g_2(t) = \phi(t) \sin w_c t$.

$= f_1 \cos w_c t \sin w_c t + f_2 \sin^2 w_c t$.

we know $2 \sin \alpha \cos \alpha = \sin 2\alpha$.

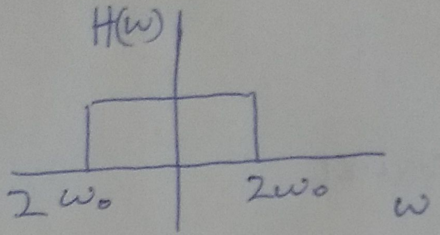
$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$.

$\Rightarrow g_2(t) = \frac{f_1}{2} \sin 2 w_c t + f_2 \left(\frac{1 - \cos 2 w_c t}{2}\right)$

$= \frac{f_2}{2} - \frac{f_2}{2} \cos(2 w_c t) + \frac{f_1}{2} \sin(2 w_c t)$

(c) As given $w_c >> w_o$

The Low Pass filter has cut off at $2w_o$



All components with w_c gets filtered out.

so $e_1 = \frac{f_1(t)}{2}$

$e_2 = \frac{f_2(t)}{2}$