① $e^{-|t|} \overset{F}{\longleftrightarrow} \dfrac{2}{\omega^2+1}$

(a) $\dfrac{d}{dt} e^{-|t|}$

From Table 5·1 Differentiation Property.

$$\frac{d^n}{dt^n}[f(t)] \xrightarrow{F} (j\omega)^n F(\omega)$$

$$\therefore \quad \frac{d}{dt}[e^{-|t|}] = (j\omega)\left(\frac{2}{\omega^2+1}\right).$$

(b) $\dfrac{1}{2\pi(t^2+1)}$

From Eq 5·1

if, $\mathcal{F}(f(t)) = F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt.$

then $\mathcal{F}^{-1}(F(\omega)) = f(t) = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega.$

As $f(t) \to F(\omega)$, if func. of time exists such that $F(t) = F(\omega)\big|_{\omega=t}$ then $\mathcal{F}\{F(t)\} = 2\pi f(-\omega)$ For $f(-\omega) = f(t)\big|_{t=-\omega}$

$\therefore$

if

$f(t) = e^{-|t|}$

$F(\omega) = \frac{2}{\omega^2+1}$

$F(t) = \frac{2}{t^2+1}$

given func. to solve is :

$$\mathcal{F}\left(\frac{1}{2\pi(t^2+1)}\right) = \frac{2}{2}\mathcal{F}\left(\frac{1}{2\pi(t^2+1)}\right)$$

$$= \frac{1}{4\pi}\mathcal{F}\left(\frac{2}{t^2+1}\right)$$

$$= \frac{1}{4\pi}\mathcal{F}(F(t))$$

AS show earlier $\mathcal{F}(F(t)) \to 2\pi f(-\omega)$. where $f(\omega) = e^{-|\omega|}$

$\therefore \frac{1}{4\pi}\mathcal{F}(F(t)) \to \frac{2\pi}{4\pi} e^{-|-\omega|}$

$$\boxed{= \frac{1}{2} e^{-|\omega|}}$$

(2)

$$g_2(t) = \begin{cases} -4 \sin(100\pi t) & 0 \le t \le 10ms. \\ 0, & \text{otherwise} \end{cases}$$

Acc. to Hint $\boxed{\text{There is Confusion in the hint } rect\left(\frac{t-5ms}{10ms}\right)}$

$$g_2(t) = rect\left[\frac{t-5ms}{10ms}\right](-4\sin(100\pi t))$$

$$\Rightarrow f(t) = rect\left[\frac{t-5ms}{10ms}\right]$$

$$g_2(t) = f(t)(-4\sin(100\pi t))$$

$$\Rightarrow A\sin\omega t = A\left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right]$$

$$g_2(t) = A\left[\frac{f(t)\, e^{j\omega t}}{2j} - \frac{f(t) e^{-j\omega t}}{2j}\right] \Rightarrow \frac{-2}{j}\left(f(t) e^{j100\pi t} - f(t) e^{-j100\pi t}\right)$$

From 5.1

① $\mathcal{F}[f(t) e^{j\omega_0 t}] \rightarrow F(\omega-\omega_0)$

From 5.2.

$\mathcal{F}(rect(t/T)) \rightarrow T\,sinc(\omega T/2)$

Also as $\mathcal{F}[f(t-t_0)] \rightarrow F(\omega)\, e^{-j\omega t_0}$ using Time transformation 5.14

$$f(t) = rect\left(\frac{t-5ms}{10ms}\right)] = rect(100t-0.5),\ \mathcal{F}(rect(100t-0.5)) = \left(\frac{1}{100}\, sinc\frac{\omega}{200}\right) e^{-\frac{0.5 j\omega}{100}}$$

$$G_2(\omega) = \frac{2}{j}\left(F(\omega-\omega_0) - F(\omega+\omega_0)\right)$$

$$= \frac{-0.02\, e^{-0.005 j\omega}}{j}\left\{ sinc[0.005(\omega+100\pi)] + sinc[0.005(\omega-100\pi)]\right]$$

additional notes for problem 2

$g_2(t) = -4 \sin 100\pi t \cdot \text{rect}\left(\frac{t - 5ms}{10ms}\right)$

$\Rightarrow g_2(t) = f_1(t) \cdot f_2(t)$; where, $f_1(t) = -4 \sin 100\pi t$

$f_2(t) = \text{rect}\left(\frac{t-5ms}{10ms}\right)$

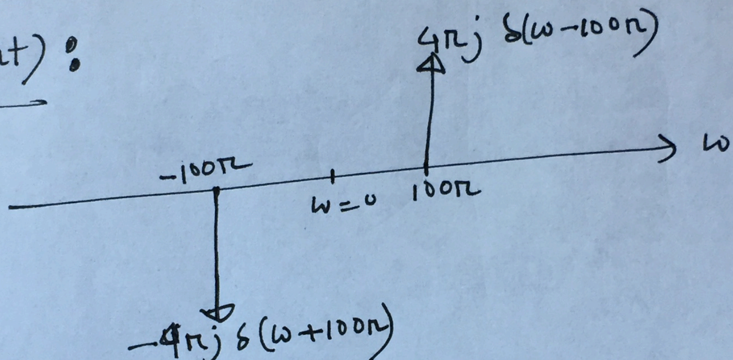
now, $f_1(t) = -4 \sin 100\pi t$

$\Rightarrow -4 \sin 100\pi t \xleftrightarrow{\mathcal{F}} -4\left[\frac{\pi}{j}\delta(\omega-\omega_0) - \frac{\pi}{j}(\omega+\omega_0)\right]$;

$\omega_0 = 100\pi$

$\Rightarrow \boxed{-4 \sin 100\pi t \xleftrightarrow{\mathcal{F}} 4\pi j\,\delta(\omega - 100\pi) - 4\pi j\,\delta(\omega + 100\pi t)}$

$\mathcal{F}(-4 \sin 100\pi t)$:



again, $f_2(t) = \text{rect}\left(\frac{t-5ms}{10ms}\right)$

$= h(t - 5ms)$; where, $h(t) = \text{rect}(t/10ms)$

now, $h(t) = \text{rect}(t/10ms) \xleftrightarrow{\mathcal{F}} T\,\text{sinc}(T\omega/2)$; $T = 10ms$

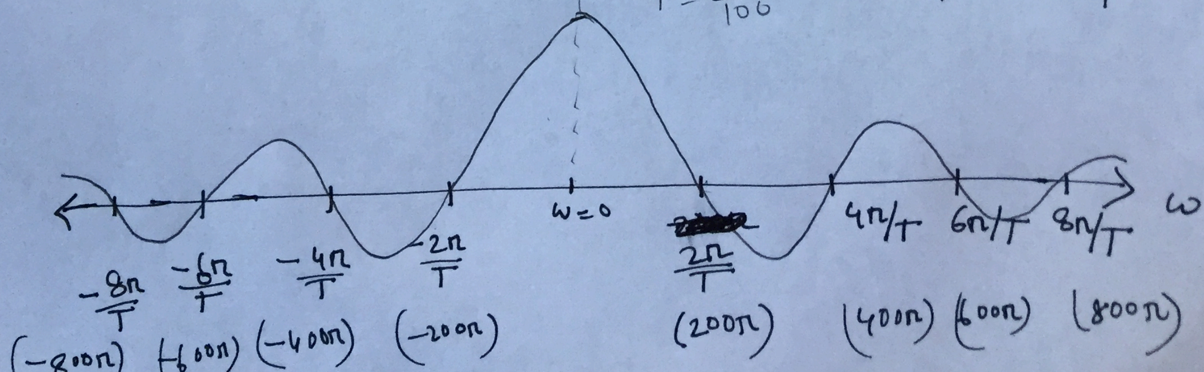
$= \frac{10}{1000}s$

$= \frac{1}{100}s$

$\Rightarrow \text{rect}(t/10ms) \xleftrightarrow{\mathcal{F}} \frac{1}{100}\,\text{sinc}\left(\frac{\omega}{200}\right)$

$= H(\omega)$

$T = \frac{1}{100}$



we know from time shift property of F.T.

$$f(t-t_0) \xleftrightarrow{\mathcal{F}} F(\omega)\, e^{-j\omega t_0}.$$

So, $f_2(t) = h(t-5ms)$

$$\Rightarrow h(t-5ms) \xleftrightarrow{\mathcal{F}} H(\omega)\, e^{-j\omega t_0}$$

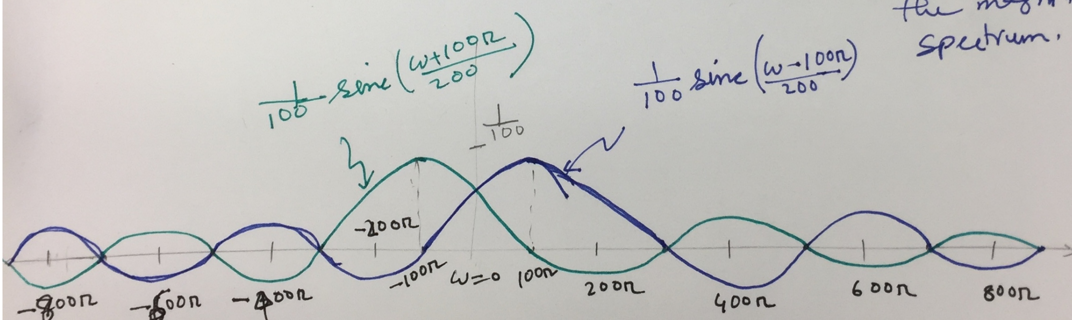
$$\Rightarrow rect\left(\frac{t-5ms}{10ms}\right) \xleftrightarrow{\mathcal{F}} \frac{1}{100}\, sinc\left(\frac{\omega}{200}\right) e^{-j\omega \cdot \frac{5}{1000}}$$

$$\Rightarrow rect\left(\frac{t-5ms}{10ms}\right) \xleftrightarrow{\mathcal{F}} \frac{1}{100}\, sinc\left(\frac{\omega}{200}\right) \left(e^{-\frac{j\omega}{200}}\right)$$

$$\mathcal{F}[f_1(t) \cdot f_2(t)] = \frac{1}{2\pi} \mathcal{F}(f_1(t)) * \mathcal{F}(f_2(t))$$

$$*\mathcal{F}\left[-4 \sin 100\pi t \cdot rect(t/10ms)\right] = \frac{1}{2\pi}\left[\frac{4\pi j}{100}\, sinc\left(\frac{\omega-100\pi}{200}\right) - \frac{4\pi j}{100}\, sinc\left(\frac{\omega+100\pi}{200}\right)\right]$$

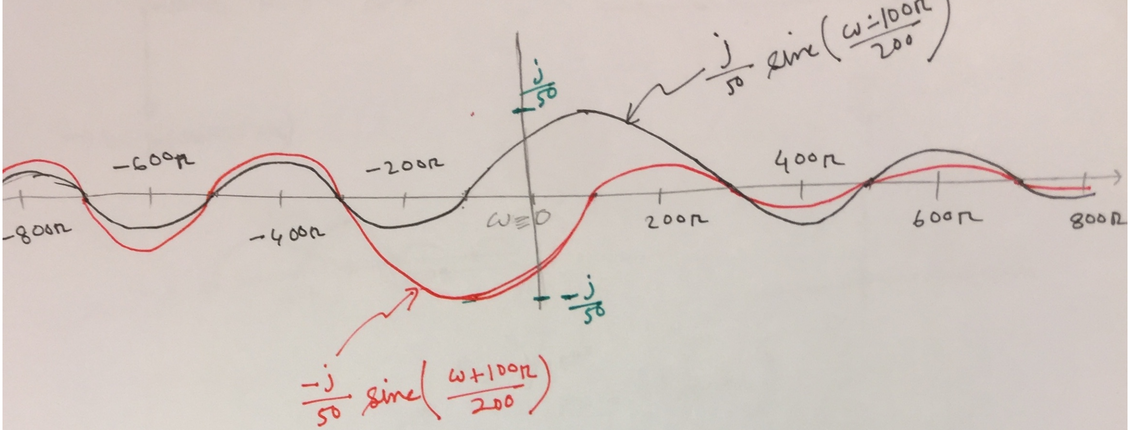
only contributes a phase. does not change the magnitude spectrum.



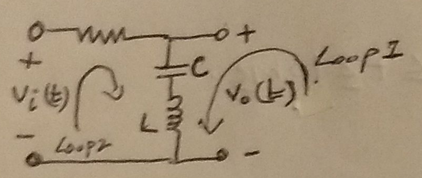
And $-4 \sin 100\pi t \cdot rect\left(\frac{t-5ms}{10ms}\right)$

$$\xleftrightarrow{\mathcal{F}} \frac{1}{2\pi}\left(\frac{4\pi j}{100}\, sinc\left(\frac{\omega-100\pi}{200}\right) e^{-\frac{j\omega}{200}} - \frac{4\pi j}{100}\, sinc\left(\frac{\omega+100\pi}{200}\right) e^{-\frac{j\omega}{200}}\right)$$

$$\Rightarrow \boxed{-4 \sin 100\pi t \cdot rect\left(\frac{t-5ms}{10ms}\right) = \frac{j}{50}\, sinc\left(\frac{\omega-100\pi}{200}\right) e^{-\frac{j\omega}{200}} - \frac{j}{50}\, sinc\left(\frac{\omega+100\pi}{200}\right) e^{-\frac{j\omega}{200}}}$$


$\frac{j}{50}$ sinc$\left(\frac{\omega - 100\pi}{200}\right)$
$\frac{j}{50}$
-600π
-200π
400π
-800π
-400π
$\omega = 0$
200π
600π
800π
$-\frac{j}{50}$
$\frac{-j}{50}$ sinc$\left(\frac{\omega + 100\pi}{200}\right)$

(3)



To Calculate freq. Resp. of filter we Cal. Transfer Function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}.$$

For $V_o(\omega)$, apply KVL at o/p. Loop 1

$$-V_o(t) + V_C(t) + V_L(t) = 0.$$

$$V_o(t) = V_C(t) + V_L(t)$$

$$V_o(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

Take FT for intial condition $I(0) = 0$

$$\boxed{V_o(\omega) = j\omega L\, I(\omega) + \frac{1}{j\omega C} I(\omega)}$$

For $V_i(\omega)$, apply KVL at I/p Loop 2.

$$-V_i(t) + R\,i(t) + L\frac{d\,i(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

Again take FT for intial condition $I(0) = 0$.

$$\boxed{V_i(\omega) = R I(\omega) + j\omega L\, I(\omega) + \frac{1}{j\omega C} I(\omega)}$$

$\therefore \quad H(w) = \dfrac{jwL\,I(w) + \frac{1}{jwC} I w}{R\,I(w) + jwL\,I(w) + \frac{1}{jwC} I(w)}$

Taking $I(w)$ common & cannel it

$$H(w) = \frac{jwL + \frac{1}{jwC}}{R + jwL + \frac{1}{jwC}}$$

$$= \frac{(j^2w^2LC + 1)/\cancel{jwC}}{(jwRC + j^2w^2LC + 1)/\cancel{jwC}}$$

$$= \frac{1 - w^2LC}{jwRC + (1 - w^2LC)}$$

$$= \frac{1}{\frac{jwRC + (1 - w^2LC)}{(1 - w^2LC)}}$$

$$H(w) = \frac{1}{1 + j\left(\frac{wRC}{1 - w^2LC}\right)}$$

The circuit in above figure 6.6 is a typical band Reject filter. To verfy this: $(1 + j\alpha)$ is the pole.

Put $\left|\dfrac{wRC}{1 - w^2LC}\right| = 1$, Plug in some conviniant values of RLC

Solve for w, Plot Magnitude Response for range of values of w. You will see Band Reject.

You get corna frequency