EE103 Lecture 29, Dec. 8 2017

- Qz 8 Average = 7.6 $\sigma = 1.9$
- Course review today (continued)
- Final Exam Dec 12 (T) 4 – 7 pm
- 2 pages of formulas & tables
- In the final exam paper, write down
  (A) Your quiz scores (8) and delete 2 and the average of the best 6 scores
  (b) Midterm score (adjusted)
  * This is to ensure that all your scores are correctly reflected in the Final Grade
  20% (Quiz) + 30% (Midterm) + 50% (Final)

Course Review on Signals & Systems

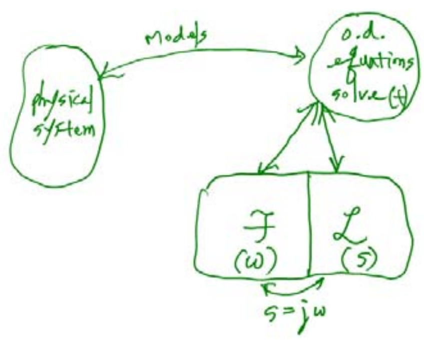
Chap 7 Laplace Transform $F(s)=\int_0^{\infty} f(t)e^{-st}dt$ $F_B(s)=\int_{-\infty}^{\infty} f(t)e^{-st}dt$

Chap 5, 6 Fourier Transform & Apps $F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \leftrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{+j\omega t}d\omega$

Chap 4 Fourier Series – Periodic signals

Chap 3 Continuous L.T.I. systems $\delta(t) \rightarrow \boxed{H} \rightarrow h(t)$; $x(t) \rightarrow h(t)*x(t)$

Chap 2 signals – even, odd-parts; periodic, non-periodic; $\delta(t)$, $u(t)$, $r(t)$, $rect(t/T)$



Time Domain — $\mathcal{F}$ — $\mathcal{L}$

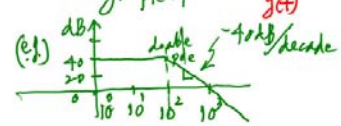
O.D.E.
$\frac{d^n}{dt^n}y(t) + a_1\frac{d^{n-1}}{dt^{n-1}}y + \cdots = \cdots$ $\quad (s^n + a_1 s^{n-1} + \cdots)Y(s) = \cdots$

$y(t) = h(t) * x(t)$ $\quad Y(\omega) = H(\omega)X(\omega)$ $\quad Y(s) = H(s)X(s)$

$= x(t) * h(t)$ $\quad H(\omega) = |H(\omega)|\underline{/H(\omega)}$ $\quad$ Partial fraction $\downarrow \mathcal{L}^{-1}$ $y(t)$

$20\log_{10}|H(\omega)|$

(e.g.)

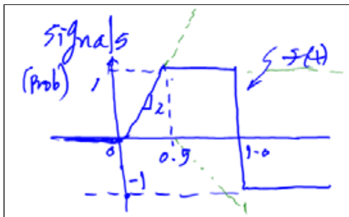


Prep. for Final
1) All HWs (10)
2) All Quizzes (8)
3) Midterm
4) Final Exam of Sp. 2017
   To be discussed/solved by TAs

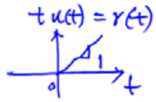
	Time-	Freq.-	s-domain
R (i, +v−)	$v = Ri$, $i = \frac{v}{R} = Gv$	$V(\omega) = R\,I(\omega)$	$V(s) = R\,I(s)$
L (i, i(0), +v−)	$v = L\frac{di}{dt}$, $i = \frac{1}{L}\int_{-\infty}^{t} v(t)dt$	$V(\omega) = j\omega L\,I(\omega)$, $I(\omega) = \frac{V(\omega)}{j\omega L}$	$V(s) = sL\,I(s)$, $I(s) = \frac{V(s)}{sL}$
C (i, +v−, v(0−))	$i = C\frac{dv}{dt}$, $v = \frac{1}{C}\int_{-\infty}^{t} i(t)dt$	$I(\omega) = j\omega C\,V(\omega)$, $V(\omega) = \frac{1}{j\omega C}I(\omega)$	$I(s) = sC\,V(s)$, $V(s) = \frac{I(s)}{sC}$

Signals

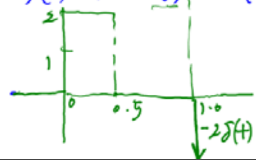
(Prob)



$t\,u(t) = r(t)$

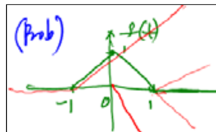


$f(t) = 2t\,u(t) - 2(t-0.5)\,u(t-0.5) - 2\,u(t-1)$



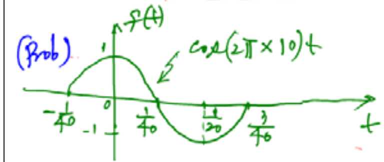
$f(t) = 2[u(t) - u(t-0.5)]$
$- 2\delta(t-1)$

(Prob)



$f(t) = (t+1)\,u(t+1) - 2t\,u(t) + (t-1)\,u(t-1)$

(Prob)



$f(t) = \cos(2\pi\times 10)t \cdot \mathrm{rect}\left((t-\tfrac{1}{40})\,\middle|\,\tfrac{1}{10}\right)$

(Prob.) $\mathcal{F}\left[\cos(2\pi\times 10)t \cdot \mathrm{rect}\left((t-\tfrac{1}{40})\,\middle|\,\tfrac{1}{20}\right)\right]$

$= \frac{1}{2\pi}\mathcal{F}[\cos(2\pi\times 10)t] * \mathcal{F}\left[\mathrm{rect}(t-\tfrac{1}{40})\,\middle|\,\tfrac{1}{20}\right]$

$= \frac{1}{2\pi}\,\pi\left[\delta(\omega-20\pi) + \delta(\omega+20\pi)\right)$

$\times e^{-j\omega\frac{1}{40}}$

$* \frac{1}{20}\,\mathrm{sinc}\left(\omega\frac{1}{40}\right)$

$\mathcal{F}[\cos\omega_0 t]$
$= \pi(\delta(\omega-\omega_0) + \delta(\omega+\omega_0))$

$\mathcal{F}[\mathrm{rect}(t|T)] = T\,\mathrm{sinc}(\omega T/2)$

$\mathcal{F}[A\,\delta(t-t_0)] = A\,e^{-j\omega t_0}$

$\int f(t-t_1)\,e^{-j\omega t}\,dt$
$= \int f(t-t_1)\,e^{-j\omega(t-t_1+t_1)}\,d(t-t_1)$
$= \int f(\tau)\,e^{-j\omega\tau}\,d\tau \cdot e^{-j\omega t_1}$

$= \frac{1}{40}\left[\mathrm{sinc}\left(\frac{\omega-20\pi}{40}\right) + \mathrm{sinc}\left(\frac{\omega+20\pi}{40}\right)\right]$

$e^{-j(\omega-20\pi)\frac{1}{40}}$ $\quad e^{-j(\omega+20\pi)\frac{1}{40}}$

$\mathcal{F}[u(t)] = \pi\,\delta(\omega) + \frac{1}{j\omega}$

For $f(t) = \begin{cases} \sin\omega_0 t & t \geq 0 \\ 0 & t < 0 \end{cases}$

$f(t) = \sin\omega_0 t\;u(t)$

$\mathcal{F}[f(t)] = \frac{1}{2\pi}\,\mathcal{F}[\sin\omega_0 t] * \mathcal{F}[u(t)]$

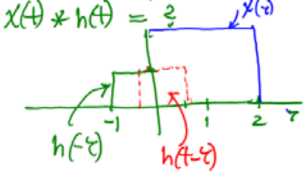
$= \frac{1}{2\pi}\,\frac{\pi}{j}\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right] *$
$\left(\pi\,\delta(\omega) + \frac{1}{j\omega}\right)$

$= \frac{\pi}{2j}\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$

$- \frac{1}{\cancel{2}}\underbrace{\left(\frac{1}{\omega-\omega_0} + \frac{1}{\omega+\omega_0}\right)} = \frac{\cancel{2}\omega}{\omega^2-\omega_0^2}$

Convolution

$x(t) * h(t) =$



i) $0 < t < 1$
$x*h = 2t$

ii) $1 < t < 2$
2

iii) $t > 2$
$2-(t-1) = 3-t$

iv) $t > 3$
$= 0$

Bode plot

$Y(j\omega) = H(j\omega)\,X(j\omega)$

$p(t) = v(t)\,i(t)$
$= v(t)\,\frac{v(t)}{R}$
$= \frac{1}{R}v^2(t)$

Power

$|Y(j\omega)|^2 = |H(j\omega)|^2\,|X(j\omega)|^2$

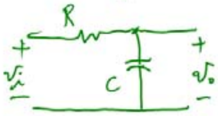
$10\log_{10}|Y(j\omega)|^2 = 10\log_{10}|H(j\omega)|^2 + 10\log_{10}|X(j\omega)|^2$

$20\log_{10}|Y(j\omega)|$

$10\log_{10}|H(j\omega)|^2 = \boxed{20\log_{10}|H(j\omega)|}$

Example

$$H(j\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$



$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\left(\frac{1}{RC}\right)}}$$

$$\frac{1}{RC} = \omega_p$$

case with $R = 1\,M\Omega$, $C = 1\,pF$

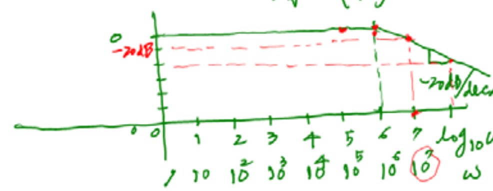
$$\frac{1}{RC} = \frac{1}{10^6(10^{-12})} = 10^6 = \omega_p$$

$$H(j\omega) = \frac{1}{1 + \frac{j\omega}{10^6}}$$

$$20\log_{10}|H(j\omega)| = 20\log_{10}\left|1 \cdot \frac{1}{\sqrt{1 + \left(\frac{\omega}{10^6}\right)^2}}\right|$$

$$= 20\log_{10} 1 + 20\log_{10}\left[1 + \left(\frac{\omega}{10^6}\right)^2\right]^{-\frac{1}{2}}$$

$$= 0 - 20\log_{10}\sqrt{1 + \left(\frac{\omega}{10^6}\right)^2}$$



(Example)

$$H(s) = 100\,\frac{(s+100)}{(s+10)(s+1000)}$$

$$H(j\omega) = 100\,\frac{(j\omega+100)}{(j\omega+10)(j\omega+1000)}$$

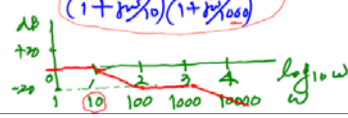
$$= 100\,\frac{100(1 + j\omega/100)}{10(1 + j\omega/10)\;1000(1 + j\omega/1000)}$$

zero

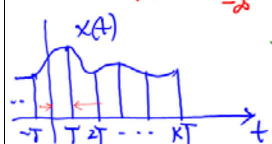
pole 1

pole 2

$$= 1\,\frac{(1 + j\omega/100)}{(1 + j\omega/10)(1 + j\omega/1000)}$$



$$\delta_T(t) = \sum_{-\infty}^{\infty}\delta(t - kT),\quad T = \text{sampling interval}$$



$$\hat{x}(t) = x(t) \times \delta_T(t)$$

$\downarrow \mathcal{F}$

$$\hat{X}(\omega) = X(\omega) * \Delta_T(\omega)\,\frac{1}{2\pi}$$

$$\mathcal{F}[\delta_T(t)] = \mathcal{F}\left[\sum_{-\infty}^{\infty}\delta(t - kT)\right] = \omega_s\sum_{-\infty}^{\infty}\delta(\omega - k\omega_s)$$

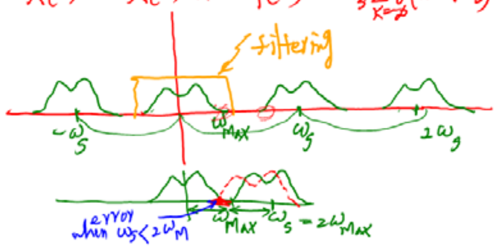
$$\omega_s = \frac{2\pi}{T}$$

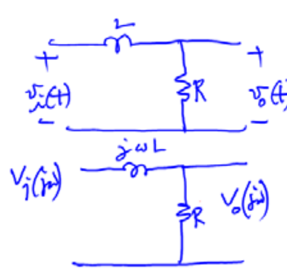
$\omega_s > 2\,\omega_{max}$ of $x(t)$

Suppose

If sampling was done with $\omega_s > 2\omega_{max}$

$$\hat{X}(\omega) = X(\omega) * \Delta_T(\omega) = \omega_s\sum_{k=-\infty}^{\infty}\delta(\omega - k\omega_s)$$





$$V_o(j\omega) = H(j\omega)\,V_i(j\omega)$$

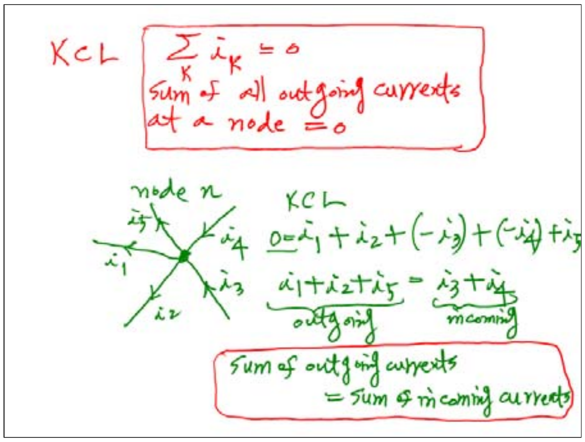
$$= \left[\frac{R}{R + j\omega L}\right] V_i(j\omega)$$

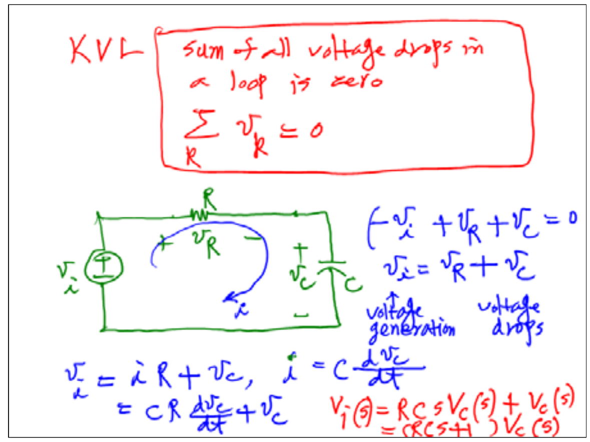
For $v_i(t) = V\cos\omega_0 t$

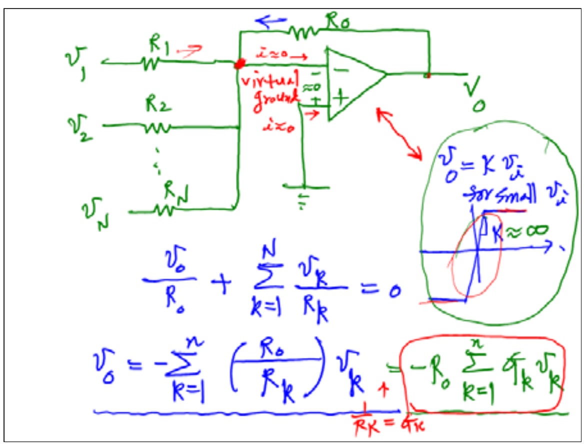
$$\boxed{v_o(t) = |H(j\omega_0)| \times V \times \cos(\omega_0 t + \angle H(j\omega_0))}$$

- $|H(j\omega_0)| = \dfrac{R}{\sqrt{R^2 + (\omega_0 L)^2}}$
- $\angle H(j\omega_0) = -\tan^{-1}\left(\dfrac{\omega_0 L}{R}\right)$

$$\frac{Z_1}{Z_2} = \frac{|Z_1|e^{j\theta_1}}{|Z_2|e^{j\theta_2}} = \left(\frac{|Z_1|}{|Z_2|}\right)e^{j(\theta_1 - \theta_2)}$$


KCL
$\sum_k i_k = 0$
sum of all outgoing currents at a node = 0
node n
i_1 i_2 i_3 i_4 i_5
KCL
$0 = i_1 + i_2 + (-i_3) + (-i_4) + i_5$
$\underbrace{i_1 + i_2 + i_5}_{\text{outgoing}} = \underbrace{i_3 + i_4}_{\text{incoming}}$
sum of outgoing currents = sum of incoming currents


KVL
sum of all voltage drops in a loop is zero
$\sum_k v_k = 0$
R
v_R
v_i
v_C
C
i
$-v_i + v_R + v_C = 0$
$v_i = v_R + v_C$
voltage generation
voltage drops
$v_i = iR + v_C, \quad i = C\frac{dv_C}{dt}$
$= CR\frac{dv_C}{dt} + v_C$
$V_i(s) = RCsV_c(s) + V_c(s) = (RCs+1)V_c(s)$


v_1 R_1
v_2 R_2
v_N R_N
R_o
$i \approx 0$
virtual ground
≈ 0
$i \approx 0$
v_o
$v_o = K v_i$
for small v_i
$K \approx \infty$
$\frac{v_o}{R_o} + \sum_{k=1}^{N} \frac{v_k}{R_k} = 0$
$v_o = -\sum_{k=1}^{n} \left(\frac{R_o}{R_k}\right) v_k = -R_o \sum_{k=1}^{n} \sigma_k v_k$
$\frac{1}{R_k} = \sigma_k$