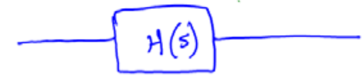
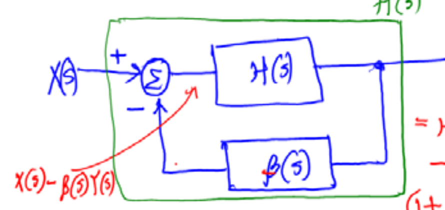
EE103 Lecture 28, Dec. 6 2017

- course review on Friday, Dec. 8 (& Dec. 6)
- Final Exam Dec 12 (T) 4 – 7 pm
- 2 pages of formulas & tables
- In the final exam paper, write down
  (a) Your quiz scores (8) and delete 2 and the average of the best 6 scores
  (b) midterm score (adjusted)
  * This is to ensure that all your scores are correctly reflected in the Final Grade
  20% (Quiz) + 30% (midterm) + 50% (Final)

Feedback Control



with feedback



$= H(s)[X(s) - \beta(s)Y(s)]$

$(1+\beta(s)H(s))Y(s) = H(s)X(s)$

Thus $\hat{H}(s) = \dfrac{H(s)}{1+\beta(s)H(s)}$

example

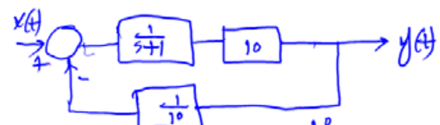
$H(s) = \frac{10}{s+1}$

$h(t) = 10e^{-t} \times u(t)$

$\hat{H}(s) = \dfrac{\frac{10}{s+1}}{1 + 5\frac{10}{s+1}} = \dfrac{10}{(s+1)+50}$

$= \dfrac{10}{s+51}$

$\hat{h}(t) = 10e^{-51t}u(t)$



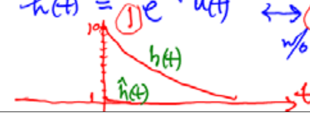
$\hat{H}(s) = \dfrac{\frac{10}{s+1}}{1+\frac{1}{10}\left(\frac{10}{s+1}\right)} = \dfrac{10}{10(s+1)+10}$

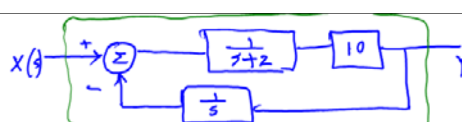
$= \dfrac{10}{10s+20} = \dfrac{1}{s+2}$

$\downarrow \mathcal{L}^{-1}$

$\hat{h}(t) = 1e^{-2t}u(t) \leftrightarrow 10e^{-t}u(t)$

w/o feedback





$\hat{H}(s) = \dfrac{\frac{10}{s+2}}{1+\left(\frac{1}{s}\right)\frac{10}{s+2}} = \dfrac{10s}{s(s+2)+10}$

$= \dfrac{10s}{s^2+2s+10} = \dfrac{10s}{(s+1)^2-(3j)^2}$

$= \dfrac{10s}{(s+1+3j)(s+1-3j)} = 10\left[\dfrac{\left(\frac{-1-3j}{-6j}\right)}{s+1+3j} + \dfrac{\left(\frac{-1+3j}{6j}\right)}{s+1-3j}\right]$

$= 10\left(\dfrac{K_1}{s+1+3j} + \dfrac{K_1^*}{s+1-3j}\right)$

$K_1 = \frac{1}{2} - \frac{1}{6}j$

$s^2+2s+10 = (s+1)^2 + 3^2 \quad (s+1)^2 - (j3)^2$

$\alpha^2+\beta^2 = (\quad)(\quad)$ ✗

$\alpha^2-\beta^2 = (\alpha+\beta)(\alpha-\beta)$ ✓

$$\xrightarrow{\mathcal{L}^{-1}} \hat{h}(t) = 10\left(K_1 e^{-t} e^{-j3t} + K_1^* e^{-t} e^{+3jt}\right)$$

$$= 10 e^{-t}\left[K_1 e^{-j3t} + \left(K_1 e^{-j3t}\right)^*\right]$$

$$= 10 e^{-t}\left[\left(\tfrac{1}{2} - j\tfrac{1}{6}\right) e^{-j3t} + \left(\tfrac{1}{2} + j\tfrac{1}{6}\right) e^{+j3t}\right]$$

$$= 10 e^{-t}\, \tfrac{1}{6}\sqrt{10}\left[\underbrace{e^{+j\tan^{-1}(-\frac{1}{3})} e^{-j3t}}_{e^{-j(3t+\tan^{-1}\frac{1}{3})}} + \text{complex conj}\right]$$

$$= \frac{5\sqrt{10}}{3} e^{-t}\left(e^{-j(3t+\tan^{-1}\frac{1}{3})} + e^{+j(3t+\tan^{-1}\frac{1}{3})}\right)$$

$$= \frac{5}{3}\sqrt{10}\, e^{-t} \cdot 2\cos\left(3t + \tan^{-1}\tfrac{1}{3}\right)$$

$$= \frac{10\sqrt{10}}{3} e^{-t} \cos\left(3t + \tan^{-1}\tfrac{1}{3}\right)$$

$\mathcal{L}_B$ bilateral Laplace Transformation

$f(t) = e^{-a|t|}$

$$\mathcal{L}_B f(t) = \int_{-\infty}^{0} e^{+at} e^{-st}\,dt + \int_{0}^{\infty} e^{-at} e^{-st}\,dt$$

$$= \int_{-\infty}^{0} e^{-(s-a)t}\,dt + \int_{0}^{\infty} e^{-(s+a)t}\,dt$$

$$= \left.\frac{e^{-(s-a)t}}{-(s-a)}\right|_{-\infty}^{0} + \left.\frac{e^{-(s+a)t}}{-(s+a)}\right|_{0}^{\infty}$$

$$= \frac{1 - e^{-(s-a)(-\infty)}}{-(s-a)} + \frac{e^{-(s+a)\infty} - 1}{-(s+a)} = \frac{1}{-(s-a)} + \frac{1}{(s+a)} = \frac{-2a}{s^2 - a^2}$$

$\Downarrow$ $Re(s-a) < 0$ or $Re\, s < a$

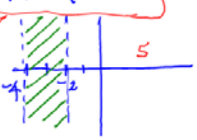
$\Downarrow$ $Re(s+a) > 0$ or $Re\, s > -a$

Find $h(t)$ for

$$H(s) = \frac{s+1}{(s+4)(s+2)}, \quad \boxed{-4 < Re\, s < -2}$$

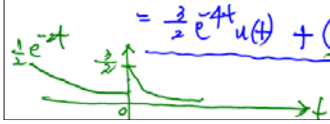
$$= \frac{3/2}{s+4} + \frac{-1/2}{s+2}$$

$Re(s+4) > 0$ $\quad$ $Re(s+2) < 0$



$$\mathcal{L}_B^{-1} H(s) = \mathcal{L}_B^{-1}\left[\frac{3/2}{s+4} + \frac{-1/2}{s+2}\right]$$

$$= \frac{3}{2} e^{-4t} u(t) + \left(+\frac{1}{2} e^{-2t} u(-t)\right)$$



$$\frac{s+1}{(s+4)(s+2)} \xrightarrow{\mathcal{L}^{-1}} \frac{2/3}{s+4} + \frac{-1/2}{s+2}$$

$Re(s+4) > 0$ $\quad$ $Re(s+2) > 0$

$\boxed{Re\, s > -2}$

$$\downarrow \mathcal{L}^{-1}$$

$$\left(\frac{2}{3} e^{-4t} - \frac{1}{2} e^{-2t}\right) u(t)$$

$$F(s) = \frac{s+4}{(s+1)(s+2)} \qquad -2 < Re\, s < -1$$

$$= \frac{3}{s+1} + \frac{-2}{s+2}$$

$\downarrow$ $Re(s+1) < 0$ $\quad$ $\downarrow$ $Re(s+2) > 0$

$$f(t) = -3 e^{-t} u(-t) + \left(-2 e^{-2t} u(t)\right)$$

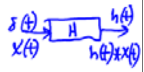
Course Review on Signals & Systems

Chap 7 — Laplace Transform $F(s)=\int_{0}^{\infty} f(t)e^{-st}dt$ $\quad F_B(s)=\int_{-\infty}^{\infty} f(t)e^{-st}dt$

Chap 5, 6 — Fourier Transform & Apps $F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \leftrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{+j\omega t}d\omega$

Chap 4 — Fourier Series – Periodic signals

Chap 3 — Continuous LTI systems



Chap 2 — Signals – Periodic, non-Periodic, even, odd-parts
$\delta(t)$, $u(t)$, $r(t)$, $\mathrm{rect}(t|T)$

Time Domain $\quad \mathcal{F} \quad \mathcal{L}$

O.D.E

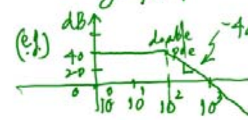
$\frac{d^n}{dt^n}y(t)+a_1\frac{d^{n-1}}{dt^{n-1}}y+\cdots=\cdots$ $\qquad (s^n+a_1s^{n-1}+\cdots)Y(s)=\cdots$

$y(t)=h(t)*x(t)$ $\qquad Y(\omega)=H(\omega)X(\omega)$ $\qquad Y(s)=H(s)X(s)$

$=x(t)*h(t)$ $\qquad H(\omega)=|H(\omega)|\angle H(\omega)$ $\qquad$ Partial fraction $\downarrow \mathcal{L}^{-1}$ $y(t)$

$20\log_{10}|H(\omega)|$

(e.g.)



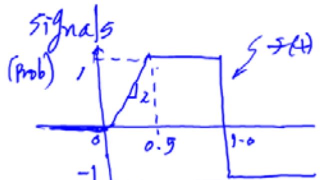
Prep. for Final

1) All HWs (10)
2) All Quizzes (8)
3) Mid-term
4) Final Exam of Sp. 2017
   To be discussed/solved by TAs

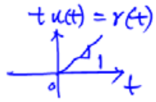
	Time-	$\mathcal{F}$req.-	S-domain
R	$v=Ri$ $i=\frac{v}{R}=Gv$	$V(\omega)=RI(\omega)$	$V(s)=RI(s)$
L (i, $i(0_-)$)	$v=L\frac{di}{dt}$ $i=\frac{1}{L}\int_{-\infty}^{t}v(\tau)d\tau$	$V(\omega)=j\omega L I(\omega)$ $I(\omega)=\frac{V(\omega)}{j\omega L}$	$V(s)=sLI(s)$ $I(s)=\frac{V(s)}{sL}$
C ($v(0_-)$)	$i=C\frac{dv}{dt}$ $v=\frac{1}{C}\int_{-\infty}^{t}i(\tau)d\tau$	$I(\omega)=j\omega C\cdot V(\omega)$ $V(\omega)=\frac{1}{j\omega C}I(\omega)$	$I(s)=sCV(s)$ $V(s)=\frac{I(s)}{sC}$

Signals

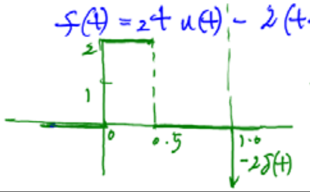
(Prob)



$t\,u(t)=r(t)$

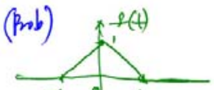


$f(t)=2t\,u(t)-2(t-0.5)u(t-0.5)-2u(t-1)$



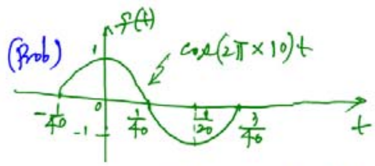
$f'(t)=2[u(t)-u(t-0.5)]-2\delta(t-1)$

(Prob)



$f(t)=(t+1)u(t+1)-2t\,u(t)+(t-1)u(t-1)$

(Prob)



$f(t)=\cos(2\pi\times10)t\cdot\mathrm{rect}\left(\left(t-\frac{1}{40}\right)\Big|\frac{1}{10}\right)$

(Prob.) $\mathcal{F}\left[\cos(2\pi\times 10)t \cdot \mathrm{rect}\left((t-\frac{1}{40})\middle|\frac{1}{20}\right)\right]$

$$= \frac{1}{2\pi}\mathcal{F}\left[\cos(2\pi\times 10)t\right] * \mathcal{F}\left[\mathrm{rect}(t-\tfrac{1}{40})\middle|\tfrac{1}{20}\right]$$

$$= \frac{1}{2\pi}\pi\left[\delta(\omega-20\pi)+\delta(\omega+20\pi)\right) * \frac{1}{20}\mathrm{sinc}\left(\omega\frac{1}{40}\right)\right]$$

$\mathcal{F}[\cos\omega_0 t]$
$= \pi\left(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right)$

$\mathcal{F}\left[\mathrm{rect}(t|T)\right] = T\,\mathrm{sinc}(\omega T/2)$

$\mathcal{F}[A\,\delta(t-t_0)] = A\,e^{-j\omega t_0}$

$$= \frac{1}{40}\left[\mathrm{sinc}\left(\frac{\omega-20\pi}{40}\right)+\mathrm{sinc}\left(\frac{\omega+20\pi}{40}\right)\right]$$

$\mathcal{F}\left[u(t)\right] = \pi\delta(\omega)+\frac{1}{j\omega}$

For $f(t) = \begin{cases}\sin\omega_0 t & t \geq 0\\ 0 & t < 0\end{cases}$

$f(t) = \sin\omega_0 t\,u(t)$

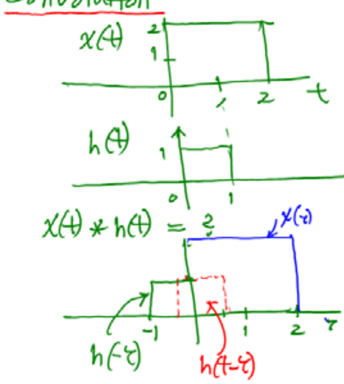
$$\mathcal{F}\left[f(t)\right] = \frac{1}{2\pi}\mathcal{F}\left[\sin\omega_0 t\right]*\mathcal{F}\left[u(t)\right]$$

$$= \frac{1}{2\pi}\frac{\pi}{j}\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right] * \left(\pi\delta(\omega)+\frac{1}{j\omega}\right)$$

$$= \frac{\pi}{2j}\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right)$$

$$= \frac{1}{2}\underbrace{\left(\frac{1}{\omega-\omega_0}+\frac{1}{\omega+\omega_0}\right)}=\frac{\omega}{\omega^2-\omega_0^2}$$

Convolution



i) $0 < t < 1$

$x*h = 2t$

ii) $1 < t < 2$

2

iii) $t > 2$

$2-(t-1) = 3-t$

iv) $t > 3$ $= 0$