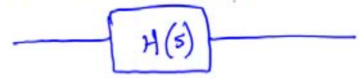
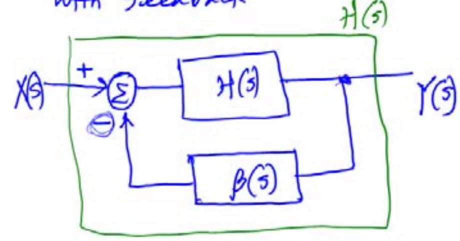
EE103 Lecture 27, Dec 4, 2017

- Qz 8 today (20 minutes)
- course review on Friday, Dec. 8 (& Dec. 6)
- HW #10 posted today

Feedback Control



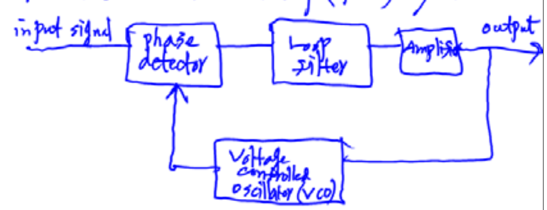
with feedback

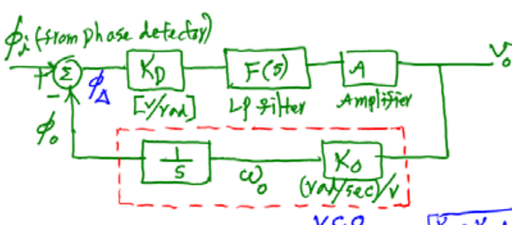


Harold Stephen Black

Born	April 14, 1898 Leominster, Massachusetts
Died	December 11, 1983 (aged 85) Summit, New Jersey
Nationality	United States
Alma mater	Worcester Polytechnic Institute
Known for	negative feedback
	Scientific career
Fields	Electrical engineer

Phase-Locked Loop (PLL) system





$\boxed{K_v = K_D A K_o}$

$$\frac{V_o}{\phi_i(s)} = \frac{K_D A F(s)}{1 + K_D A F(s) \frac{K_o}{s}}$$

where $\omega_o(t) = \frac{d}{dt}\phi_o(t)$ $[\phi_o(s) = \frac{1}{s}\omega_o(s)]$

$\omega_i(s) = s\,\phi_i(s)$

$$\frac{V_o(s)}{\omega_i(s)} = \frac{1}{s}\frac{V_o(s)}{\phi_i(s)} = \frac{K_D A F(s)}{s + K_D A F(s) K_o}$$

Time Domain Model

$\phi_\Delta(t) = f_1(\phi_i(t))\ f_2(\phi_o(t))$ output of phase detector

$f_1(\phi_i(t))$ = input to phase detector

$f_2(\phi_o(t))$ = output of VCO

$\frac{d\phi_o(t)}{dt} = \omega_o(t) = \omega_{free} + K_o v_o(t)$

K_o = sensitivity [Hz/V]

LP filter

$$\frac{dx}{dt} = Ax + b\,\phi_\Delta(t)$$
$$= Ax + b\,f_1(\phi_i)\,f_2(\phi_o)$$
$$= Ax + b\,\gamma(\phi_i - \phi_o)$$
$$\frac{d\phi_o}{dt} = \omega_{free} + K_o(Ax)$$

Example

let $f_1(\phi_i) = A_1 \sin(\phi_i(t))$

$f_2(\phi_o) = A_2 \cos(\phi_o(t))$

where $\phi_i(t) = \omega_i(t) + \theta_i(t)$

For LP RC filter

$$\dot{x} = -\frac{1}{RC}x + \frac{1}{RC} A_1 A_2 \sin\phi_i(t)\cos\phi_o(t)$$

& $\dot{\phi}_o = \omega_{free} + K_o(Ax)$

the phase detector characteristics $\Rightarrow$

$$A_1 A_2 \sin\phi_i(t)\cos\phi_o(t)$$
$$= \tfrac{1}{2} A_1 A_2 \sin(\phi_i(t) - \phi_o(t))$$
$$\triangleq \gamma(\phi_i - \phi_o)$$

$$\Rightarrow \boxed{\begin{aligned} \dot{x} &= -\frac{1}{RC}x + \frac{1}{RC}\frac{A_1A_2}{2}\sin\phi_\Delta(t) \\ \dot{\phi}_o &= \omega_{free} + K_o(Ax) \end{aligned}}$$

where $\phi_\Delta = \phi_i - \phi_o$

Application (Numerical Example)

case w/o LP filter

$$\frac{V_o(s)}{\phi_i(s)} = \frac{K_v \frac{1}{K_o}}{s + K_v}$$

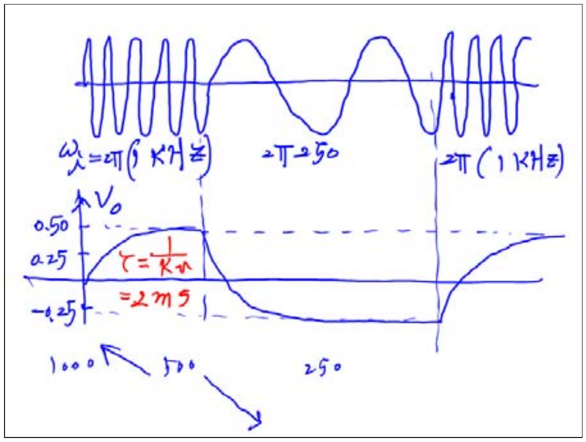
let $K_o = 2\pi\left[\frac{1\ KHz}{V}\right]$

$K_v = 500/\text{sec}$

$\omega_{free} = 500$ HZ

$$V_o = \frac{\omega_i - \omega_o}{K_o} = \frac{\omega_\Delta}{K_o}$$

ω_i	V_o
$2\pi \times$ 250 HZ	$V_o = \frac{2\pi(250) - 2\pi(500)}{2\pi\left(\frac{1\ KHz}{V}\right)} = -0.25$ V
$2\pi \times$ 1 KHZ	$V_o = \frac{2\pi(1000) - 2\pi(500)}{2\pi\left(\frac{1\ KHz}{V}\right)} = +0.5$ V



Case with

$$\phi_i(t) = 2\pi(500\,Hz)\left(1 + 0.1\sin(2\pi\times100)t\right)$$

$$\frac{V_o(j\omega)}{\phi_i(j\omega)} = \frac{1}{K_o}\frac{K_v}{s+K_v}\Big|_{s=j\omega}$$

$$= \frac{1}{K_o}\frac{K_v}{K_v+j\omega}$$

$$= \frac{1}{2\pi(1\,KHz/V)}\;\underbrace{\frac{500}{500+j2\pi(100)}}_{=0.39-j0.48}$$

$(\omega = 2\pi\times100)$

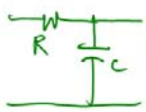
$$= 0.05(0.62\angle -51^\circ)$$

corresponding

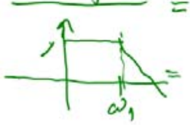
$$v_o(t) = 0.031\sin(2\pi\times100t - 51^\circ)$$

Case with a RC filter F(s)

$$\frac{V_o(s)}{\phi_i(s)} = \frac{\frac{1}{K_o}K_v F(s)}{s + K_v F(s)} \qquad (A)$$



$$F(s) = \frac{1/Cs}{R + 1/Cs} = \frac{1}{1+RCs}$$



$$= \frac{1}{1+\frac{s}{(1/RC)}} = \frac{1}{1+\frac{s}{\omega_1}} \qquad (B)$$

$(\omega_1 = 1/RC)$

From (A) & (B)

$$\frac{V_o(s)}{\phi_i(s)} = \frac{\frac{1}{K_o}K_v\left(\frac{1}{1+s/\omega_1}\right)}{s + \left(\frac{1}{1+s/\omega_1}\right)K_v}$$

$$= \frac{1}{K_o}\frac{K_v}{K_v + s\left(1+\frac{s}{\omega_1}\right)}$$

$$= \frac{1}{K_o}\frac{K_v}{K_v + s + \frac{s^2}{\omega_1}}$$

$$= \frac{1}{K_o}\frac{K_v\omega_1}{s^2 + \omega_1 s + K_v\omega_1}$$

o.d.e

$\downarrow s$

$$(s^3 + 3s^2 + 4s + 2)Y(s) = s X(s)$$

$$\frac{d^3}{dt^3}y + 3\frac{d^2}{dt^2}y + 4\frac{dy}{dt} + 2y = \frac{dx}{dt}$$

LHS RHS

$$s^3 + 3s^2 + 2s + 2s + 2$$

$$= s(s^2+3s+2) + 2(s+1)$$

$$= s(s+2)(s+1) + 2(s+1)$$

$$= (s+1)[s(s+2)+2]Y(s) = s X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{s}{(s+1)(s^2+2s+2)}$$

$$s^2+2s+2 = (s+1)^2 - j^2 = (s+1+j)(s+1-j)$$

$$\frac{K_1}{s+1} + \frac{K_2}{s+1+j} + \frac{K_2^*}{s+1-j}$$

$$\frac{s}{(s+1)(s+1+j)(s+1-j)}$$

$\times(s+1)\big|_{s=-1}$ $K_1 = ?$ $\quad \frac{-1}{j(-j)} = \frac{-1}{-j^2} = -1$

$K_2 = ?$

$\times(s+1+j)\big|_{s=-1-j}$ $\quad \frac{-1-j}{(-j)(-2j)} = \frac{-1-j}{-2} = \frac{1}{2} + \frac{1}{2}j$

K_2^* $\quad \frac{1}{2} - \frac{1}{2}j$

$$h(t) \overset{u(t)}{=} -1\,e^{-t} + \left(\frac{1}{2}+\frac{1}{2}j\right)e^{-(1+j)t} + \left(\frac{1}{2}-\frac{1}{2}j\right)e^{-(1-j)t}$$

$$f(t) \xleftrightarrow{\mathcal{L}} F(s)$$

$$f(\infty) = \lim_{s \to 0} \left[sF(s) - f(0_-) \right]$$

$$f(t) = \cos \omega_0 t \xleftrightarrow{\mathcal{L}} \frac{\omega_0}{s^2 + \omega_0^2}$$

$$\cos(\omega_0 \infty) \neq 0 \leftrightarrow \lim_{s \to 0} s \frac{\omega_0}{s^2 + \omega_0^2} = 0$$

not application