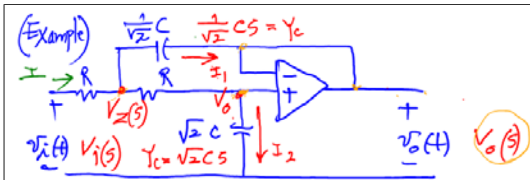
EE103 Lecture 24, Nov. 27, 2017

- Qz 7 Avg = 9.03, $\sigma = 1.55$
- HW 9 posted
- Qz 8 on Dec 4, next Monday
  HW 10 will be posted on Dec 4, but no more Qz.
- course Review on Dec. 8

(Example)



$V_i(s) = RI + V_Z(s)$, $\quad I = I_1 + I_2$

$$V_Z(s) = R\left(\sqrt{2}Cs\,V_o(s)\right) + V_o(s)$$

$$I = \left[V_Z(s) - V_o(s)\right]\frac{1}{\sqrt{2}}Cs + \sqrt{2}Cs\,V_o(s)$$

$$V_i(s) = R\left[\left(R\sqrt{2}Cs\,V_o + V_o - V_o\right)\frac{1}{\sqrt{2}}Cs + \sqrt{2}Cs\,V_o\right] + R\sqrt{2}Cs\,V_o + V_o$$

$$= R\left[R(Cs)^2 V_o + \sqrt{2}Cs\,V_o\right] + R\sqrt{2}Cs\,V_o + V_o$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{R^2C^2s^2 + 2\sqrt{2}RCs + 1}$$

$$= \frac{1}{R^2C^2}\,\frac{1}{s^2 + 2\frac{\sqrt{2}}{RC}s + \frac{1}{R^2C^2}}$$

$$= \frac{1}{R^2C^2}\,\frac{1}{\left(s + \frac{\sqrt{2}}{RC}\right)^2 - \frac{1}{R^2C^2}}$$

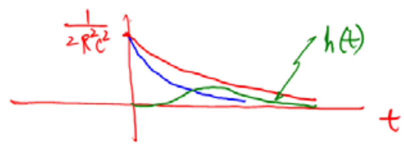
$$= \frac{1}{R^2C^2}\,\frac{1}{\left(s + \frac{\sqrt{2}}{RC} + \frac{1}{RC}\right)\left(s + \frac{\sqrt{2}}{RC} - \frac{1}{RC}\right)}$$

$$= \frac{1}{R^2C^2}\,\frac{1}{\left(s + \frac{\sqrt{2}+1}{RC}\right)\left(s + \frac{\sqrt{2}-1}{RC}\right)}$$

$$= \frac{1}{2R^2C^2}\left(\frac{-1}{s + \frac{\sqrt{2}+1}{RC}} + \frac{1}{s + \frac{\sqrt{2}-1}{RC}}\right)$$

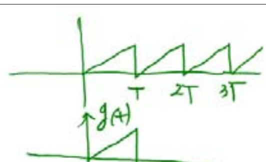
$\downarrow \mathcal{L}^{-1}$

$$h(t) = \frac{1}{2R^2C^2}\left(e^{-\frac{(\sqrt{2}-1)}{RC}t} - e^{-\frac{(\sqrt{2}+1)}{RC}t}\right)u(t)$$



Time Periodicity

$f(t) = f(t+T)$



$f(t) = g(t) * \delta_T(t)$
$k \geq 0$

$\downarrow \mathcal{L}$

$$F(s) = G(s)\left[1 + e^{-sT} + e^{-2sT} + \cdots\right]$$

$$= G(s)\,\frac{1}{1 - e^{-sT}}$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$= 1 + x\left(1 + x + x^2 + \cdots\right)$$

$$= 1 + x\left(\frac{1}{1-x}\right) = 1 + \frac{x}{1-x}$$

$$= \frac{1-x+x}{1-x} = \frac{1}{1-x}$$, where $x = e^{-sT}$

alternatively,

$f(x) = \frac{1}{1-x}$

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \cdots$$

$$= 1 + \left.\frac{1}{(1-x)^2}\right|_{x=0} x + \frac{2}{(1-x)^3}\frac{1}{2!}x^2 + \cdots$$

$$= 1 + x + x^2 + \cdots$$

Example 7.13 (page 368)

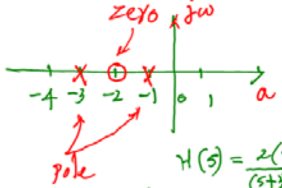
$$H(s) = \frac{4s+8}{2s^2+8s+6} = \frac{2(s+2)}{s^2+4s+3}$$

$4s+8 = 4(s+2)$

$$= \frac{2(s+2)}{(s+3)(s+1)} = \left[\frac{+1}{s+3} + \frac{+1}{s+1}\right]$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$h(t) = \left(e^{-t} + e^{-3t}\right)u(t)$$



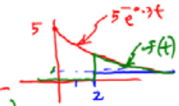
$$H(s) = \frac{2(s+2)}{(s+3)(s+1)} = \frac{\overset{1}{A}}{s+3} + \frac{\overset{1}{B}}{s+1}$$

$\times(s+3)$

$$\left.\frac{2(s+2)}{s+1}\right|_{s=-3} = A + \left.\frac{B(s+3)}{s+1}\right|_{s=-3} \quad A = +1$$

$$\times(s+1) \quad \left.\frac{2(s+2)}{s+3}\right|_{-1} = \left.\frac{1(s+1)}{s+3}\right|_{s=-1} + B$$

Example 7.5 (p 352)



$$f(t) = 5e^{-0.3t}u(t-2)$$

$$= 5e^{-0.3(t-2+2)}u(t-2)$$

$$= 5e^{-0.6}\underbrace{e^{-0.3(t-2)}u(t-2)}$$

$$\mathcal{L}\left[e^{-0.3t}u(t)\right] = \frac{1}{s+0.3}$$

$$\mathcal{L}\left[e^{-0.3(t-2)}u(t-2)\right] = e^{-2s}\left[\frac{1}{s+0.3}\right]$$

$$\text{thus } \mathcal{L}\left[5e^{-0.3t}u(t-2)\right] = \underset{2.744}{5e^{-0.6}}\left[e^{-2s}\frac{1}{s+0.3}\right]$$

$$x(t) \xrightarrow{\mathcal{L}} X(s)$$

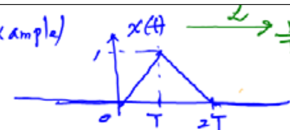
$$x'(t) = \frac{dx(t)}{dt} \xrightarrow{\mathcal{L}} sX(s) - x(0_-)$$

$$x''(t) = \frac{d}{dt}\left(\frac{dx(t)}{dt}\right) \xrightarrow{\mathcal{L}} s\left[sX(s) - x(0_-)\right] - x'(0_-)$$

$$\vdots$$

$$x^{(n)}(t) = \frac{d^n x(t)}{dt^n} = s^n X(s) - s^{n-1}x(0_-) - s^{n-2}x'(0_-) - \cdots - x^{(n-1)}(0_-)$$

(example)



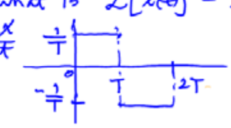
$$\xrightarrow{\mathcal{L}} \frac{1}{T}\frac{1}{s^2}\left(1 - 2e^{-sT} + e^{-2sT}\right)$$

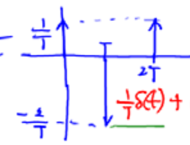
$$= \frac{1}{T}\left(\frac{1-e^{-sT}}{s}\right)^2$$

what is $\mathcal{L}[x(t)] = X(s)$?

$\uparrow \times \frac{1}{s}$

$\times \frac{1}{s}$ $\uparrow$





$$\mathcal{L} \rightarrow \frac{1}{T} - \frac{2}{T}e^{-sT} + \frac{1}{T}e^{-2sT}$$

$$\frac{1}{T}\delta(t) + \left(-\frac{2}{T}\right)\delta(t-T) + \frac{1}{T}\delta(t-2T)$$

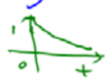
$$f(t) \xrightarrow{\mathcal{L}} F(s)$$

$$t\,f(t) \xrightarrow{\mathcal{L}} -\frac{d}{ds}F(s)$$

$$t^2 f(t) = t\left[t\,f(t)\right] \xrightarrow{\mathcal{L}} -\frac{d}{ds}\left(-\frac{d}{ds}F(s)\right)$$

$$= \frac{d^2}{ds^2}F(s)$$

(Example)



$$e^{-at}u(t) \xrightarrow{\mathcal{L}} \frac{1}{s+a}$$

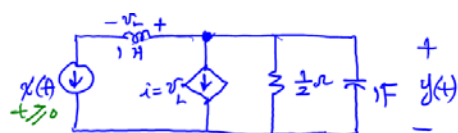
$$t\,e^{-at}u(t) \xrightarrow{\mathcal{L}} -\frac{d}{ds}\left(\frac{1}{s+a}\right) = \frac{+1}{(s+a)^2}$$

$$t^2 e^{-at}u(t) \xrightarrow{\mathcal{L}} -\frac{d}{ds}\left[\frac{1}{(s+a)^2}\right] = \frac{2}{(s+a)^3}$$

$$t^3 e^{-at}u(t) \xrightarrow{\mathcal{L}} -\frac{d}{ds}\left(\frac{2}{(s+a)^3}\right) = \frac{3\cdot 2}{(s+a)^4}$$

$$\vdots$$

$$t^n e^{-at}u(t) \xrightarrow{\mathcal{L}} \frac{n!}{(s+a)^{n+1}}$$



$y(0_-) = 2$

KCL

$$x(t) + x'(t) + 2y(t) + y'(t) = 0$$

$$\Big\downarrow \mathcal{L}$$

$$X(s) + \left[sX(s) - \cancel{x}\right] + 2Y(s) + sY(s) - 2 = 0$$

$x(0_-)$

solve for $Y(s)$

$$Y(s)\left[s+2\right] = 2 - X(s) - sX(s) + x(0_-)$$

$$= 2 - X(s)\left[s+1\right] + x(0_-)$$

$$\left\langle \frac{Y(s)}{X(s)} = -\frac{s+1}{s+2} + \frac{2}{X(s)} \right\rangle \quad Y(s) = \frac{2+x(0_-)}{s+2} - \frac{s+1}{s+2}X(s)$$

$$Y(s) = \frac{2 + x(0^-)}{s+2} - \frac{s+1}{s+2} X(s)$$

For $x(t) = e^{-3t} u(t) \xrightarrow{\mathcal{L}} \frac{1}{s+3}$

$$Y(s) = \frac{2 + x(0^-)}{s+2} - \left[\frac{s+1}{s+2} \frac{1}{s+3} \right]$$

$$= \frac{2 + x(0^-)}{s+2} - \left[\frac{-1}{s+2} + \frac{2}{s+3} \right]$$

$$\Big\downarrow \mathcal{L}^{-1}$$

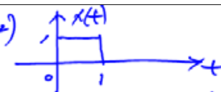
due to i.c.

$$y(t) = \left((2 + x(0^-)) e^{-2t} + e^{-2t} - 2 e^{-3t} \right) u(t)$$

$$= \left((x(0^-) + 3) e^{-2t} - 2 e^{-3t} \right) u(t)$$

$$y(0) = x(0^-) + 3 - 2 = 2 \Rightarrow \underbrace{x(0^-) = 1}_{\text{required}}$$

(example 2)



$$x(t) = u(t) - u(t-1)$$

$$\Big\downarrow \mathcal{L}$$

$$X(s) = \frac{1}{s} - e^{-1s} \frac{1}{s} = \frac{1}{s}\left(1 - e^{-s}\right)$$

$$Y(s) = \frac{2 + x(0^-)}{s+2} - \frac{s+1}{s+2} \frac{1}{s}\left(1 - e^{-s}\right)$$

$$= \frac{2}{s+2} - \frac{s+1}{(s+2)s} - \left[\frac{s+1}{(s+2)s} e^{-s} \right]$$

$$\downarrow \; f(t)u(t) \qquad \downarrow \; f(t-1)\,u(t-1)$$

$$= \frac{2 + \overset{1}{x(0^-)}}{s+2} - \left(\frac{\frac{1}{2}}{s+2} + \frac{\frac{1}{2}}{s} \right) -$$

$$y(t) = 3 e^{-2t} u(t) - \underbrace{\frac{1}{2}\left(e^{-2t} u(t) + u(t) \right)}_{f(t)} - f(t-1)$$

$y(0) = 2$ ✓