EE 103 - HW1

① The signals in Figure P2.1 are zero except as shown.

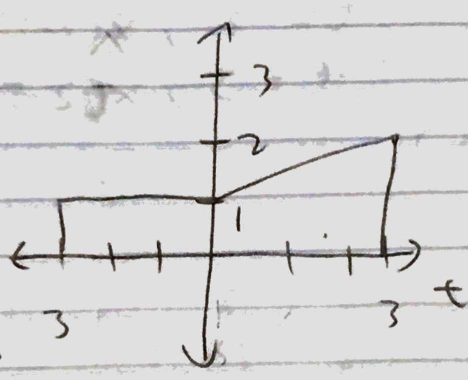
ⓐ For the signal $x(t)$ of Figure P2.1ⓐ, plot

(i) $x(-t/3)$ (ii) $x(-t)$

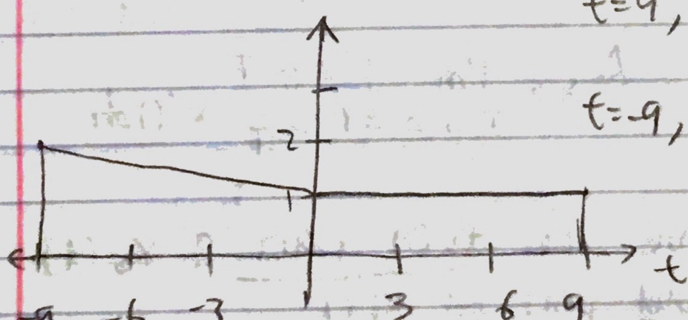
(iii) $x(3+t)$ (iv) $x(2-t)$

verify your results by checking at least two points

ⓐ



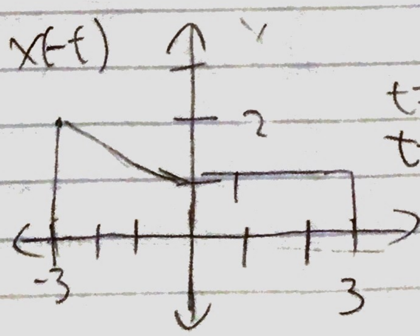
(i) $x(-t/3)$



$t=9, \; x(\frac{-9}{3}) = x(-3) = 1$

$t=-9, \; x(-(\frac{-9}{3})) = x(+3) = 2.$

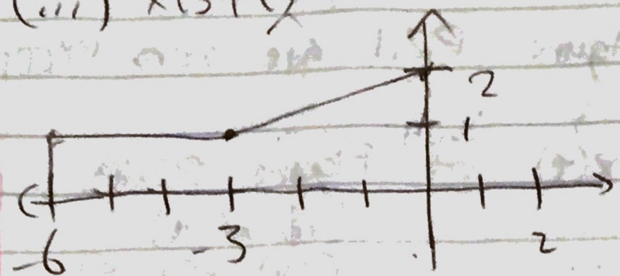
(ii) $x(-t)$



$t=3, \; x(-(3)) = 1$

$t=-3, \; x(+3) = 2.$

(iii) $x(3+t)$

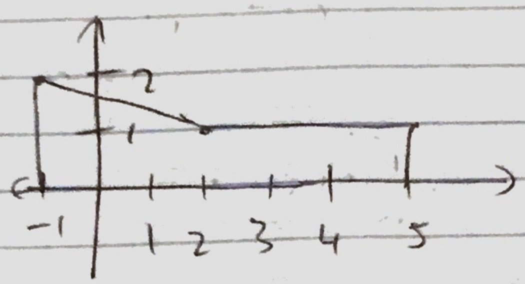


$t=0,\ x(3+0)=x(3)=2$

$t=3,\ x(3-3)=x(0)=1$

$t=6,\ x(3-6)=x(-3)=1$

(iv) $x(2-t)$



$t=-1,\ x(2+1)=x(3)=2$

$t=2,\ x(2-2)=x(0)=1$

$t=5,\ x(2-5)=x(-3)=1$

② The average value A_x of a signal $x(t)$ is given by

$$A_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\,dt$$

Let $x_e(t)$ be the even part & $x_o(t)$ be the odd part of $x(t)$

(a) show that

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x_o(t)dt = 0$$

$$\int_{-T}^{T} x_o(t)dt = \int_{-T}^{0} x_o(t)\, dt + \int_{0}^{T} x_o(t)\, dt \qquad // x_o(t) = -x_o(-t)$$

$$\Rightarrow \int_{-T}^{0} x_o(t)\, dt = -\int_{-T}^{0} x_o(-t)\, dt \Big|_{x=-\tau}$$

$$= \int_{-T}^{0} x_o(\tau)d\tau$$

$$= -\int_{0}^{T} x_o(\tau)d\tau$$

$$\Rightarrow \boxed{\int_{-T}^{T} x_o(t)dt = 0}$$

(b) show that

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x_e(t)\, dt = A_x$$

$$\int_{-T}^{T} x(t)dt = \int_{-T}^{T} [x_e(t) + x_o(t)]dt$$

$$= \int_{-T}^{T} x_e(t)\, dt \qquad // \text{ since we know } \int_{-T}^{T} x_o(t)dt = 0$$

Then

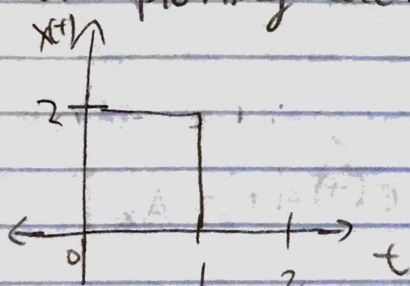
$$A_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, dt = \boxed{\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x_e(t)dt = A_x}$$

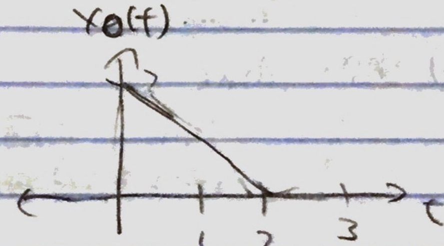
(c) Show that $x_o(0) = 0$ & $x_e(0) = x(0)$

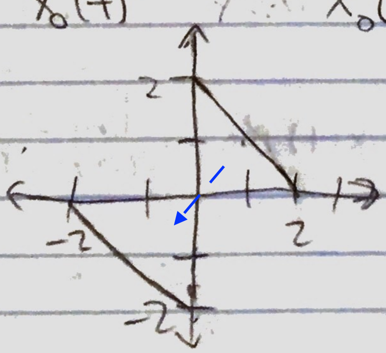
Since $x_o(0) = -x_o(-0) = -x_o(0)$,
the only number way to show that
$x_o(a) = 0$ is that when $a = 0$,
then $x(0) = x_e(0) + x_o(0)$
$= x_e(0)$

There is a typo in the question Figure P2.11
The 2nd signal is xo(t) not xe(t)

(3) Given in Figure P.2.11 are the parts of a signal $x(t)$ and its odd part $x_o(t)$, for $t \geq 0$ only; that is, $x(t)$ and $x_o(t)$ for $t < 0$ are not given. Complete the plots of $x(t)$ and $x_e(t)$, and give a plot of the even part, $x_e(t)$, of $x(t)$. Give the equations used for plotting each part of the signals







$x_o(t) = \frac{x(t) - x(-t)}{2}$

$x_o(1) = -1$

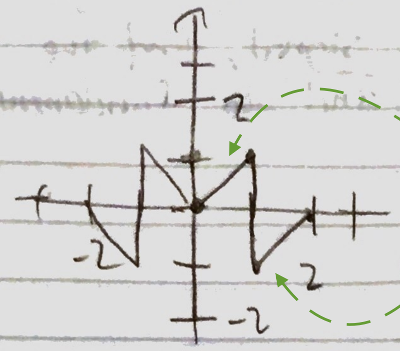
$x_o(2) = 0$

$x_o(2) = 2$

xo(-t)=-xo(t)—>symmetric w.r.t origin
=> xo(t<0) is the reflection of xo(t>0) w.r.t origin

$$X_e(t) = X(t) - X_o(-t)$$

$X_e(t)$



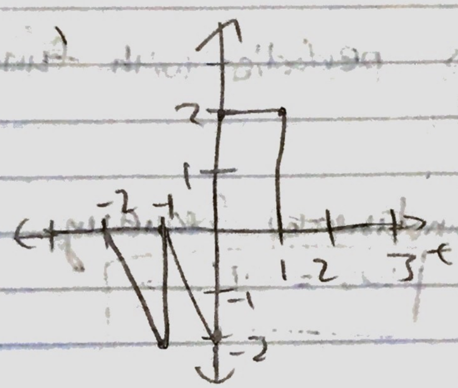
$$x_e = \frac{x(t) + x(-t)}{2}$$

for t >= 0

$t = 1_-$,	$x_e(1) = 2 - 1 = 1$	(left limit)
$t = 1_+$,	$x_e(1_+) = 0 - 1 = -1$	(right limit)
$t = 2$,	$x_e(2) = 0$	
$t = 0$,	$x_e(0) = \frac{2-2}{2} = 0$	

xe(-t)=xe(t)—>symmetric w.r.t t=0 line
=> xe(t<0) is the reflection of xe(t>0) w.r.t t=0 line

$X(t) = X_e(t) + X_o(t)$



$X(0) = 0 + (-2) = -2$

$x(0_+) = 0 + 2 = 2$

$x(1_-) = 1 + 1 = 2$

$x(1_+) = -1 + 1 = 0$

$x(2) = 0 + 0 = 0$

$x(-1_+) = 1 + (-1) = 0$

$x(-1_-) = -1 + (-1) = -2$

$x(-2) = 0 + 0 = 0$

④ Prove mathematically that the signals given are periodic. For each signal, find the fundamental period (T_0) and the fundamental frequency w_0.

ⓐ $x(t) = 7\sin 3t$

ⓑ $x(t) = \sin(8t + 30°)$

ⓐ $x(t) = 7\sin 3t$

$\Rightarrow \sin(t) = \sin(t + n2\pi)$ for any integer n,

so $7\sin(3t) = 7\sin(3t + n2\pi) = 7\sin(3(t + n\frac{2\pi}{3}))$.

$\Rightarrow$ Therefore, $x(t)$ is periodic with fundamental period $\boxed{T_0 = \frac{2\pi}{3}}$

and fundamental frequency $\boxed{w_0 = \frac{2\pi}{T_0} = 3}$

ⓑ $x(t) = \sin(8t + 30°)$

since $\sin(t) = \sin(t + n2\pi)$ for any integer n,

$\sin(8t + 30°) = \sin(8(t + \frac{2\pi}{8}) + 30°)$

$= \sin(8t + 2\pi + 30°)$

where $\boxed{w_0 = 8}$ and $\boxed{T_0 = \frac{2\pi}{8} = \frac{\pi}{4}}$