EE103 F2017 Signals and Systems

Lecture #4 Oct 9, 2017 1:20-2:25pm
Stevens 175

More on signals (Book 2.3 - 2.5)

Study Table 2.3

1. $\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$

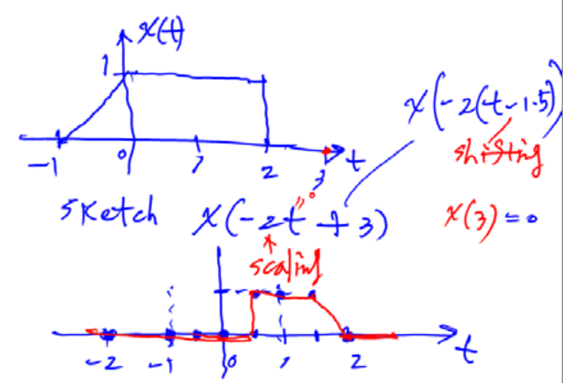
2. $\int_{-\infty}^{\infty} f(t-t_0)\,\underline{\delta(t)}\,dt = f(-t_0)$

3. $\frac{d}{dt}u(t-t_0) = \delta(t-t_0)$

⋮

6. $\int_{-\infty}^{\infty} \delta(at-t_0)\,dt = \frac{1}{|a|}\int_{-\infty}^{\infty}\delta(t-\frac{t_0}{a})\,dt$

exercise

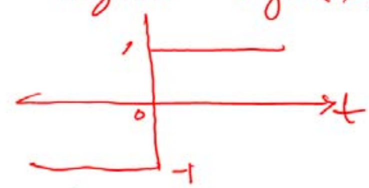


$x(-2(t-1.5))$ shifting

Sketch $x(-2t+3)$ scaling

$x(3)=0$

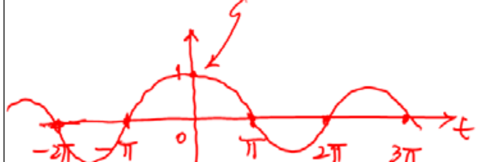
Signum $sgn(t) = \begin{cases} -1 & t<0 \\ 0 \text{ or else} & t=0 \\ +1 & t>0 \end{cases}$



relationship between $sgn(t)$ & $u(t)$

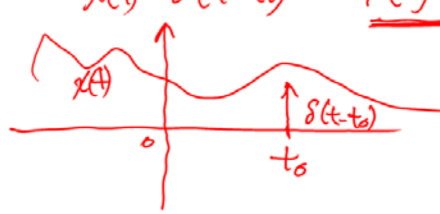
$$\boxed{u(t) = \frac{1}{2}(1+sgn(t))}$$

$sinc(t) = \frac{\sin t}{t}$



Sifting property

$x(t)\cdot\delta(t-t_0) = \underline{x(t_0)\,\delta(t-t_0)}$



$$\int_{-\infty}^{\infty} x(t)\,\delta(at)\,dt$$

$at=\tau$

$dt=\frac{1}{a}\,d\tau$

$t=\frac{\tau}{a}$

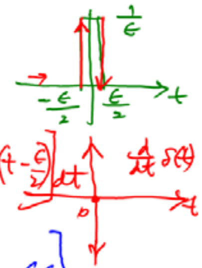
$$\int_{-\infty}^{\infty} x\left(\frac{\tau}{a}\right)\delta(\tau)\,\frac{1}{a}\,d\tau = \frac{1}{a}\int_{-\infty}^{\infty} x\left(\frac{\tau}{a}\right)\delta(\tau)\,d\tau = \frac{1}{a}\,x(0)$$

$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$$

$$\Rightarrow \delta(at)=\frac{1}{|a|}\,\delta(t)$$

$\delta(-3t)=\frac{1}{3}\delta(t)$

$$\int_{-\infty}^{\infty} x(t)\,\delta'(t)\,dt$$



$$\lim_{\epsilon\to 0}\frac{1}{\epsilon}\int_{-\infty}^{\infty} x(t)\left[\delta\left(t+\frac{\epsilon}{2}\right)-\delta\left(t-\frac{\epsilon}{2}\right)\right]dt$$

$$=\lim_{\epsilon\to 0}\frac{1}{\epsilon}\left[x\left(-\frac{\epsilon}{2}\right)-x\left(\frac{\epsilon}{2}\right)\right]$$

$$=\lim_{\epsilon\to 0}\frac{-1}{\epsilon}\left[x\left(\frac{\epsilon}{2}\right)-x\left(-\frac{\epsilon}{2}\right)\right] = -x'(0)$$

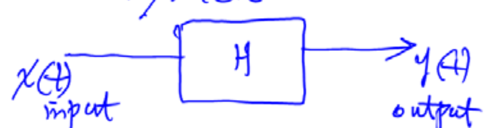
$$\int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)\,dt = \begin{cases} -x'(t_0) & t_1\le t_0\le t_2 \\ 0 & \text{elsewhere}\end{cases}$$

In general,

$\frac{d^k}{dt^k}\delta(t)$ k-th derivative

$$\int_{t_1}^{t_2} x(t)\,\delta^{(k)}(t-t_0)\,dt = \begin{cases} (-1)^k x^{(k)}(t_0) & t_1\le t_0\le t_2 \\ 0 \end{cases}$$

System



$$y(t)=H(x(t))$$

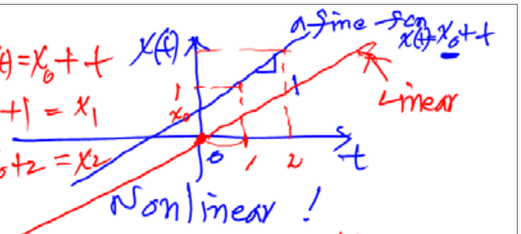
Linear Time-Invariant System

(LTI system)

$x(t)=x_0+t$

$x_0+1=x_1$

$x_0+2=x_2$



Nonlinear !

Linear Combination

$$H(ax_1+bx_2) \overset{?}{=} a\,H(x_1)+b\,H(x_2)$$

Linear