EE103 F2017 Signals and Systems

Lecture #3 Oct 9, 2017 1:20-2:25pm

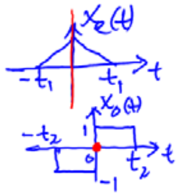
Stevens 175

More on signals (Book 2.3 – 2.5)

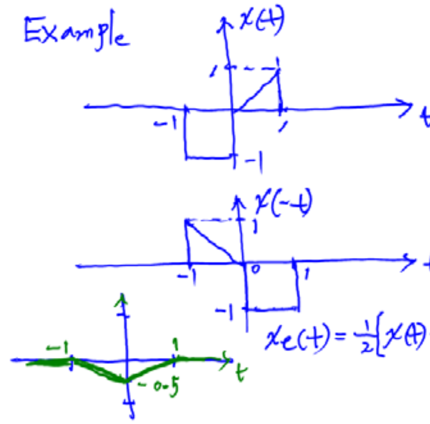
$x(t) = x_e(t) + x_o(t)$

even $x_e(t) = x_e(-t)$

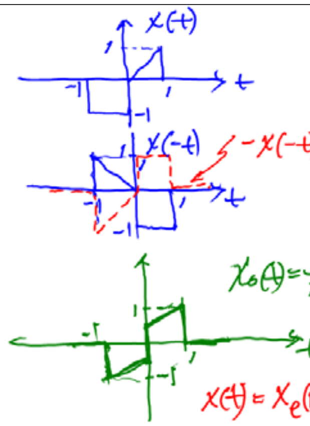
odd $x_o(t) = -x_o(-t)$



Example



$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$



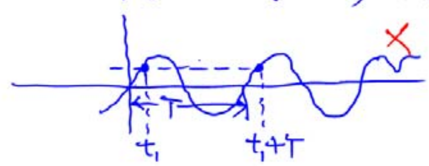
$x_o(t) = \frac{1}{2}(x(t) - x(-t))$

$x(t) = x_e(t) + x_o(t)$

Periodic & Aperiodic

$x(t)$ is a periodic signal with period T

if $x(t) = x(t + T)$ for all t.



$x_1(t) = \sin 2\pi t + \frac{2}{1+t^2}$

Is this periodic ? aperiodic

$\sin 2\pi t \stackrel{?}{=} \sin 2\pi(t + T)$

$\sin(\underset{2\pi t}{\alpha} + \underset{2\pi T}{\beta}) = \sin\alpha \underbrace{\cos\beta}_{=1} + \cancel{\cos\alpha \underbrace{\sin\beta}_{=0}}$

$v(t) = 120 \sin \omega t \quad \omega = 2\pi(60)$

$p(t)|_R = v(t) \cdot \frac{v(t)}{R}$

$= (120 \sin \omega t)^2 / R$

$= 14400 \sin^2 \omega t / R$

where $\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$

$\underset{\text{average power}}{100W} = 14400 \left[\frac{1}{2}(1 - \underbrace{\cos 2\omega t}_{\text{avg} = 0})\right] / R$

$= 7200 / R \quad \Rightarrow R = 72\Omega$

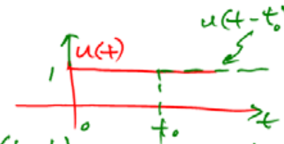
Signal Power

$$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt$$
$$= \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\,dt$$
$$= \frac{1}{RT}\int_0^T V^2 \sin^2\omega t\,dt$$
$V = 120$, $\sin^2\omega t = \frac{1}{2}(1-\cos 2\omega t)$
$$= \frac{1}{RT} V^2 \frac{1}{2}T - 0 = \frac{1}{R}\frac{V^2}{2}$$
$$V=120 \quad \frac{1}{R}\,\frac{14400}{2} = \frac{7200}{R} = 100\,W$$

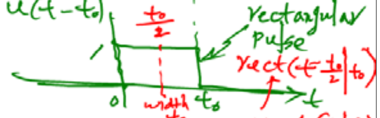
$u(t)$

$u(t-t_0)$

pulse function

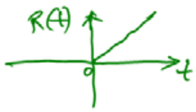


$u(t) - u(t-t_0)$



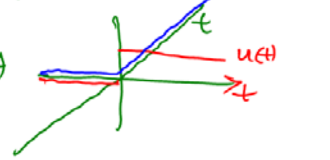
$\mathrm{rect}(t|t_0)$

Ramp function $R(t)$

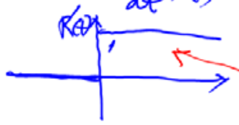


$$R(t) = \begin{cases} 0 & t<0 \\ 0 & t=0 \\ t & t>0 \end{cases}$$

$$R(t) = t\,u(t)$$



$R'(t)$

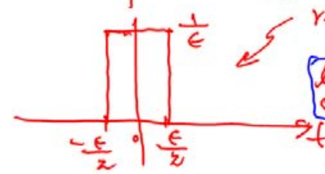


$$\frac{d}{dt}R(t) = \frac{d}{dt}\left(t\,u(t)\right)$$
$$= \frac{d}{dt}t\cdot u(t) + t\frac{du(t)}{dt}$$
$$= u(t)$$

$\delta(t)$ $\overset{0}{?}$ $t=0$ 0: $t<0$, $t>0$

Delta function (singularity) $\delta(t)$

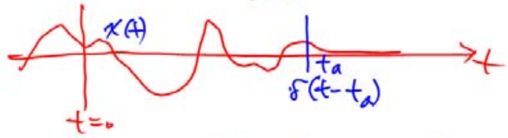
$\mathrm{rect}(t|\epsilon)$



$$\boxed{\lim_{\epsilon\to 0}\mathrm{rect}(t|\epsilon) = \delta(t)}$$

$$\boxed{\delta(-t) = \delta(t) \text{ even function}}$$

$$t\cdot\delta(t) = t\big|_{t=0}\cdot\delta(0) = 0$$



$$x(t)\cdot\delta(\underbrace{t-t_a}_{=0}) = \begin{cases} 0 & \text{everywhere except} \\ x(t_a), & t=t_a \end{cases}$$

$$\delta(t-t_a)\big|_{t=t_a} = \delta(0)$$

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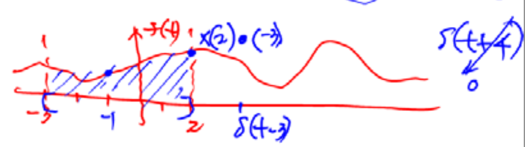
e.g.

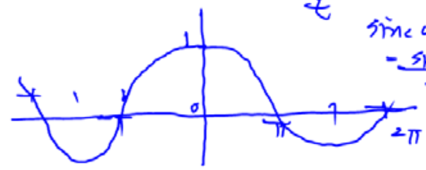
$$e^{-3t}\,\delta(t-1) = e^{-3(1)} = e^{-3}$$
$$e^{-3t}\,\delta(t-10) = e^{-3(10)}$$

$$\int_{-3}^{2} f(t)\left[2 + \delta(t+1) - 3\,\delta(t-2) + 10\,\delta(t-3)\right]dt$$
$$= \int_{-3}^{2} f(t)\,2\,dt + f(-1) - 3f(2) + 0$$



$\text{sinc}(t) = \dfrac{\sin t}{t}$

$\text{sinc}\,\omega t = \dfrac{\sin \omega t}{\omega t}$



Study Table 2.3

1. $\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$
2. $\int_{-\infty}^{\infty} f(t-t_0)\,\underline{\delta(t)}\,dt = f(-t_0)$
3. $\frac{d}{dt}u(t-t_0) = \delta(t-t_0)$

$\vdots$

6. $\int_{-\infty}^{\infty} \delta(at-t_0)\,dt = \frac{1}{|a|}\int_{-\infty}^{\infty}\delta\left(t-\frac{t_0}{a}\right)dt$

exercise