EE103 Lect #22 Nov 20, 2017

Note: Course materials (lecture slides, HWs & solutions) are posted on EE103 website:
https://ee103-fall2017-01.courses.soe.ucsc.edu/

Also webcast can be watched at
https://webcast.ucsc.edu & EE103

No quiz on Nov. 27
Lecture on Nov 22 (yes)
HW 8 assigned today → Quiz on Dec 4.

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example of $\mathcal{L}$

$$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = ?$$

$$\int_0^\infty \frac{d}{dt}f(t)\, e^{-st}\,dt$$

$$= f(t)e^{-st}\Big|_0^\infty - \int_0^\infty f(t)\, d(e^{-st})$$

$$\overset{\text{Re}\, s>0}{=} 0 - f(0)\,1 - \int_0^\infty f(t)(-s)e^{-st}\,dt$$

$$= -f(0_-) + s\underbrace{\int_0^\infty f(t)\, e^{-st}\,dt}_{=F(s)}$$

$$= sF(s) - f(0_-)$$

$$\delta(t) \xrightarrow{\mathcal{L}} \int_{0_-}^\infty \delta(t)\, e^{-st}\,dt = 1$$

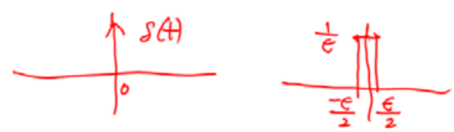
$$u(t) \xrightarrow{\mathcal{L}} \int_0^\infty u(t)e^{-st}\,dt = \int_0^\infty 1\, e^{-st}\,dt$$

$$= \frac{e^{-st}}{-s}\Big|_0^\infty = \frac{1}{s}$$

$$\frac{d}{dt}u(t) = \delta(t)$$

$$\mathcal{L}\left[\frac{d}{dt}u(t)\right] = sU(s) - u(0_-)$$

$$= s\,\frac{1}{s} - u(0_-) = 1$$



$$\int_0^\infty \delta(t)\, e^{-st}\,dt \qquad \delta(t) = \lim_{\epsilon\to\infty} \text{rect}(t|\epsilon)$$

$$\int_0^\infty \frac{d}{dt}f(t)e^{-st}\,dt = sF(s) - f(0_-)$$

$\lim_{s\to 0}$ LHS $f(\infty) - \cancel{f(0)}$

RHS $\lim_{s\to 0} sF(s) - \cancel{f(0_-)}$

$$\lim_{s\to 0} sF(s) = f(\infty)$$

(example) $f(t) = e^{-3t} \xrightarrow{\mathcal{L}} \frac{1}{s+3}$ (Re $s > 0$)

$$f(\infty) = 0$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\frac{s}{s+3} = 0 \quad \text{check}$$

For $\lim s\to\infty$

$$\int_0^\infty \frac{d}{dt}f(t)e^{-st}\,dt = sF(s) - f(0_-)$$

LHS $0 = \lim_{s\to\infty} sF(s) - f(0_-)$

$$\lim_{s\to\infty} sF(s) = f(0_-)$$

(eg) $\frac{1}{s+3}$

$$\lim_{s\to\infty} sF(s) = \frac{s}{s+3} = 1$$

$$f(t) = e^{-3t},\ f(0) = 1 \quad \text{check}$$

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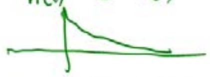
Table 7-3

$$x(t) \longrightarrow \boxed{\frac{1}{s+3}} \longrightarrow y(t)$$

$H(s)$

$h(t) = e^{-3t}u(t)$



$y(t) = x(t) * h(t)$

$Y(s) = X(s)\,H(s)$

when $x(t) = e^{-4t}u(t)$

$X(s) = \frac{1}{s+4}, \quad Y(s) = \frac{1}{(s+3)(s+4)}$

$$\mathcal{L}^{-1}\left[\frac{1}{(s+3)(s+4)}\right] = \mathcal{L}^{-1}\left[\frac{1}{s+3} + \frac{-1}{s+4}\right]$$

$\frac{A}{s+3} + \frac{B}{s+4} = \frac{1}{(s+3)(s+4)}$ $\quad A=1,\ B=-1$

$\frac{A(s+4)+B(s+3)}{(s+3)(s+4)} = \frac{1}{(s+3)(s+4)}$

$s(A+B) = 0 \Rightarrow A = -B$

$4A + 3B = 1 \rightarrow 4(-B) + 3B = 1$

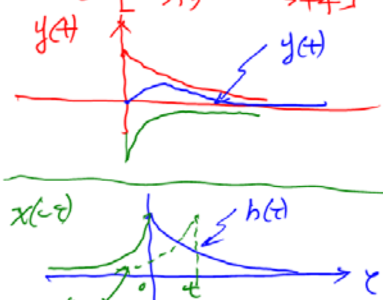
$-B = 1 \Rightarrow \boxed{B = -1}$

$A = -B = (+1)$

For A $\quad X(s+3)$

$\cancel{\frac{(s+3)}{(s+3)(s+4)}}\Big|_{s+3=0} = \frac{1}{1} = 1$

$$\mathcal{L}^{-1}\left[\frac{1}{s+3} + \frac{-1}{s+4}\right] = (e^{-3t} - e^{-4t})u(t)$$



$= (e^{-3t} - e^{-4t})u(t)$

$= e^{-4t}(e^{t} - 1)$

$$\int_{-\infty}^{t} e^{-4(t-\tau)} \cdot e^{-3\tau}\,d\tau = e^{-4t}\int_{0}^{t} e^{\tau}\,d\tau = e^{-4t}\left[e^{\tau}\right]_0^t$$

(Example)

$$\mathcal{L}[t] = \int_0^\infty t\,e^{-st}\,dt$$

$$= \int_0^\infty t\,d\left[\frac{e^{-st}}{-s}\right]$$

$$= t\left(\frac{e^{-st}}{-s}\right)\Big|_0^\infty - \int \frac{e^{-st}}{-s}\,dt$$

$$= (0-0) + \frac{1}{s}\underbrace{\int_0^\infty e^{-st}\,dt}_{=\frac{1}{s}} = \frac{1}{s^2}$$

$$\frac{d}{dt}(t) = u(t) \quad t>0 \quad \xrightarrow{\mathcal{L}} \frac{1}{s}$$

$$sF(s) - f(0) = s\frac{1}{s^2} - f(0) = \frac{1}{s}$$