

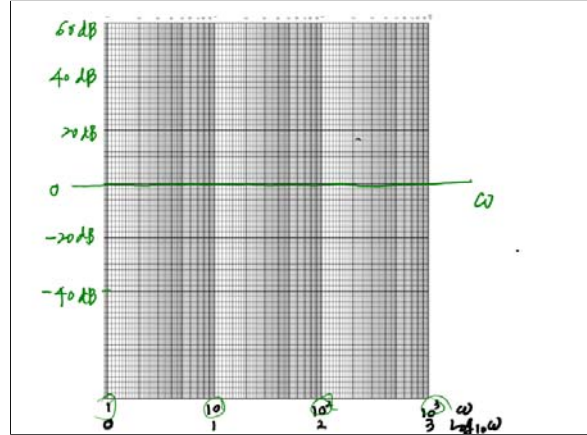
EE103 lect #19 Nov 13, 2017

HW #7 posted  
QZ 8 today

sect 6.1, 6.2, 6.3

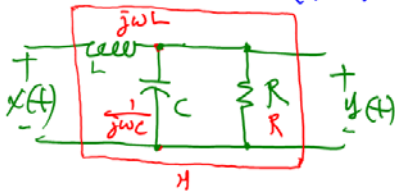
update on midterm scores

Prob 1-4 Avg 56.06 σ 21.31	prob 1-3 graded $\times \frac{100}{80}$ 66.48 σ 23.79
100 highest 90s 2 40s 11 80s 11 30s 4 70s 11 20s 5 60s 13 <20 7 50s 19	100 highest 90s 14 40s 3 80s 17 30s 5 70s 10 20s 4 60s 13 <20 4 50s 13



Nth order Butterworth Filter

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^{2N}}}$$



$$H(\omega) = \frac{\frac{1}{j\omega C} \parallel R}{j\omega L + \frac{1}{j\omega C} \parallel R}$$

$$= \frac{1}{j\omega L (j\omega C + \frac{1}{R}) + 1}$$

$$= \frac{1}{(1 - \omega^2 LC) + j\omega \frac{L}{R}}$$

$$|H(\omega)| = \frac{1}{\sqrt{(1 - \omega^2 LC)^2 + (\omega \frac{L}{R})^2}}$$

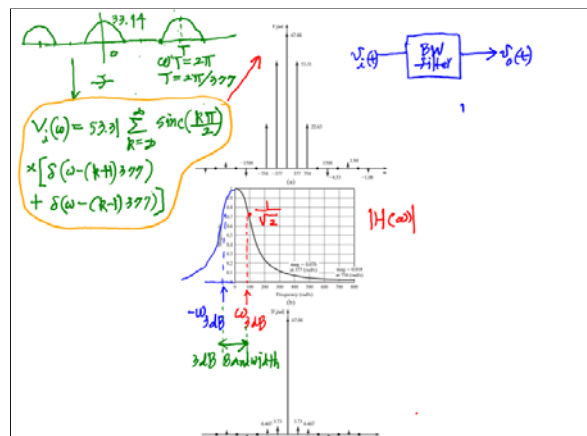
For  $L = 2R^2C$ ,

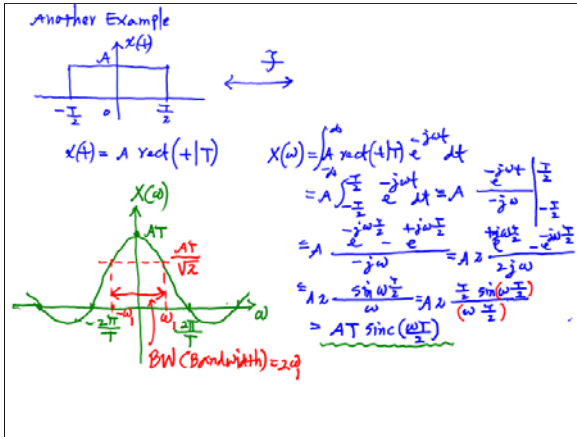
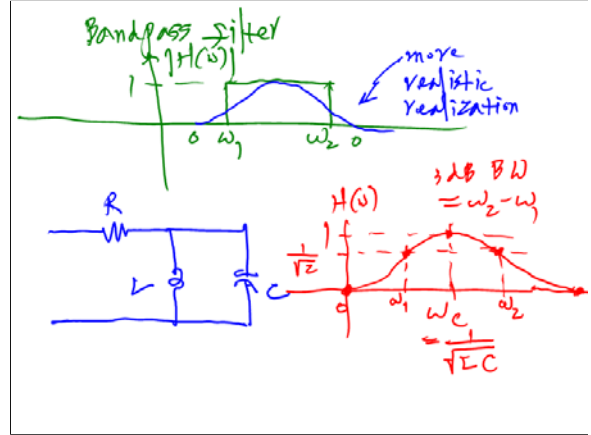
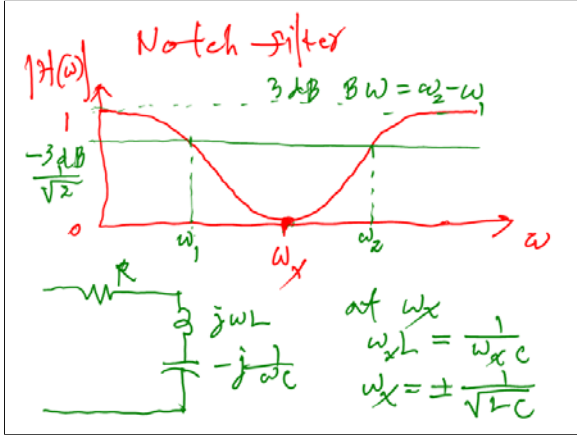
$$|H(\omega)| = \frac{1}{\sqrt{1 - 2\omega^2 LC + \omega^2(LC)^2 + \omega^2(\frac{L}{R})^2}}$$

where  $\omega^2(\frac{L}{R})^2 - 2\omega^2 LC = \omega^2 \frac{L^2}{R^2} - 2\omega^2 LC$   
 $= \omega^2 L (\frac{L}{R^2} - 2C) \stackrel{L=2R^2C}{=} \omega^2 L (\frac{2R^2C}{R^2} - 2C) = 0$

$$\Rightarrow |H(\omega)| = \frac{1}{\sqrt{1 + \omega^2(LC)^2}} = \frac{1}{\sqrt{1 + \omega^4(\frac{L}{2C})^2}} = \frac{1}{\sqrt{1 + (\omega/\omega_c)^4}}$$

$N=2$ , thus 2nd order Butterworth Filter





**Born** April 30, 1916  
Pasadena, Michigan, United States

**Died** February 24, 2001 (aged 84)  
Bedford, Massachusetts, United States

**Nationality** American

**Alma mater** University of Michigan, MIT

**Known for** [Invent](#)

**Awards for**

- Shuart Belliniere Medal (1955)
- IEEE Medal of Honor (1966)
- National Medal of Science (1965)
- Harvey Prize (1972)
- Claude E. Shannon Award (1972)
- Harold Pender Award (1976)
- John F. Maclean (1981)
- Kyoto Prize (1985)
- National Inventors Hall of Fame (2004)

**Fields** Mathematics and electronic engineering

**Institutions** Bell Labs, MIT, Institute for Advanced Study

**Theses** A symbolic analysis of relay and switching circuits

Shannon's Sampling Theorem

Let  $f(t)$  can be expressed as

$$f(t) = A_0 + \sum_{k=1}^M A_k \cos(\omega_k t + \theta_k)$$

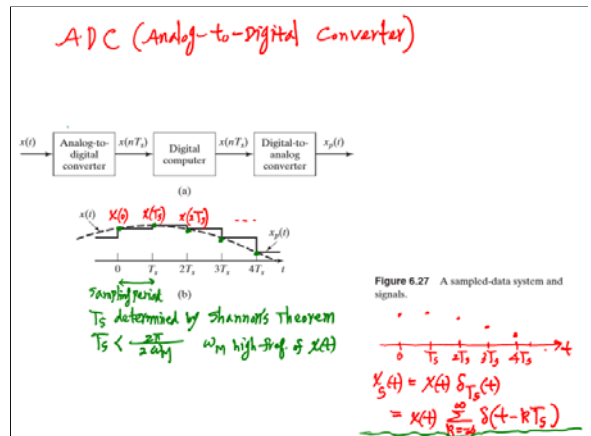
max freq  $\omega_M = M \omega_0$

According to Shannon's sampling theorem [7], to avoid aliasing and allow for complete reconstruction of the continuous signal, the sampling frequency must be greater than twice  $\omega_M$ , the highest-frequency component of the signal to be sampled; that is,  $\omega_s > 2\omega_M$ . The frequency  $2\omega_M$  is known as the Nyquist rate.

$$\Rightarrow T_s < \frac{2\pi}{2\omega_M} = \frac{\pi}{M \omega_0} = \frac{\pi}{M 2\pi f_0} = \frac{1}{2M f_0} = \frac{T_0}{2M}$$

If  $f(t) = A_1 \cos 5t + A_2 \cos 10t + A_3 \cos 50t$

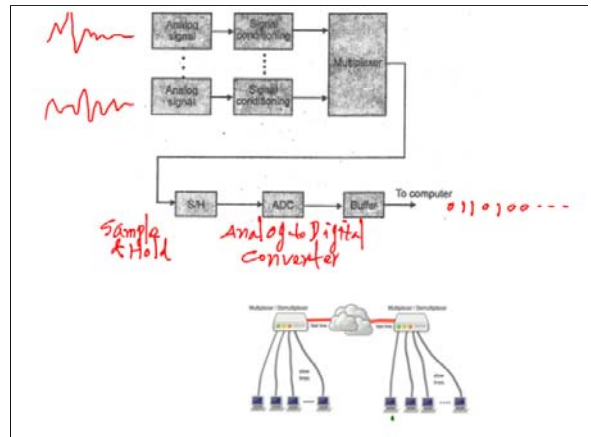
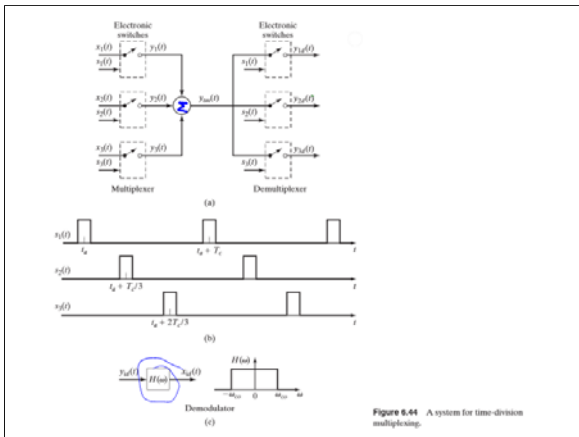
$\omega_M = 50$ ,  $\omega_s > 2\omega_M = 100$  (highest freq)

$$T_s < \frac{2\pi}{2 \times 50} = \frac{\pi \text{ rad}}{50 \text{ rad/sec}}$$


$x_s(t) = X(t) \sum_{k=0}^{\infty} \delta(t - kT_s)$   
 $\mathcal{F}[x_s(t)] = X(\omega) * \mathcal{F}[\sum_{k=0}^{\infty} \delta(t - kT_s)]$   
 Recall that  $\sum_{k=0}^{\infty} \delta(t - kT_s) = \sum_{k=0}^{\infty} C_k e^{+jk\omega_s t}$   
 where  $\omega_s = \frac{2\pi}{T_s}$   
 $C_k = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} (\sum_{k=0}^{\infty} \delta(t - kT_s)) dt = \frac{1}{T_s}$   
 then  $\sum_{k=0}^{\infty} \delta(t - kT_s) = \frac{1}{T_s} \sum_{k=0}^{\infty} e^{+jk\omega_s t}$   
 $\mathcal{F}[\sum_{k=0}^{\infty} \delta(t - kT_s)] = \frac{1}{T_s} \sum_{k=0}^{\infty} \delta(\omega - k\omega_s)$   
 $\mathcal{F}[x_s(t)] = X(\omega) * \frac{1}{T_s} \sum_{k=0}^{\infty} \delta(\omega - k\omega_s)$   
 $= \frac{1}{T_s} \sum_{k=0}^{\infty} X(\omega - k\omega_s)$   
 suppose  $X(\omega)$

Michelle  
 Tom  
 dad mom

Time sampled & time-division multiplexing  
 Frequency modulation & transmission  
 Signal reception & demodulation  
 Low pass filtering



Back to ADC

$x_s(kT_s) \rightarrow$  Binary expression  
 quantization error

$x(t) \rightarrow$  Sample  $x_s(t) \rightarrow$  ADC  $\rightarrow$  Binary sequence  $\rightarrow$  Digital Processing  $\rightarrow$  Binary sequence  $\rightarrow$  DAC  $\rightarrow$   $y_s(t)$  (Analog)

Nyquist freq.  $> 2f_s$

Signal Modulation

comm. channel  
 A carrier  $\omega_c$   
 Bandwidth  $2B$   
 L-P Filter

